Optimization of Relative Orbital Configurations for the XTI Harmony Mission

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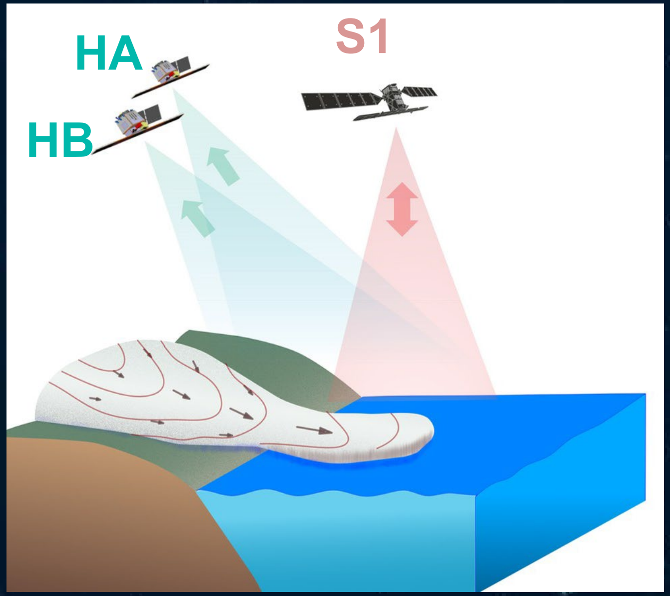
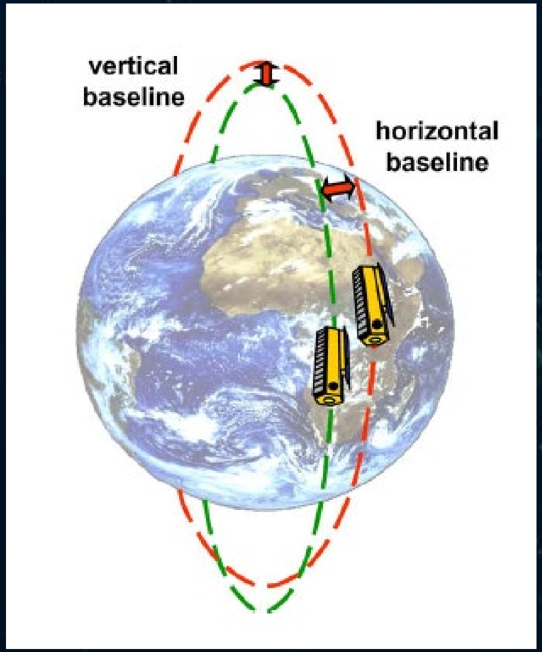
Helix Formation during the XTI phase of Harmony Mission





Definition: A 3D satellite formation that combines out-of-plane (horizontal) orbital displacements (by relative inclination vector) with in-plane (vertical) orbital displacements (by relative eccentricity vector).

Key benefit: It allows safe distances between companion satellites while keeping radar interferometric performance.

Temporal lag

- It is the delay that aligns the spectral information (support) of the two radar images of Harmony-A and Harmony-B.
- Critical, especially for studying ocean movements.

Height of Ambiguity (HoA)

- The vertical height that causes a 2π phase shift.
- Smaller HoA→higher sensitivity, but harder phase unwrapping.

$$h_{amb} = \frac{\lambda \cdot R \cdot \sin(\theta_i)}{B_\perp}$$

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Optimization for the XTI Harmony Mission Configuration

(in terms of the relative parameters Δe, Δi)

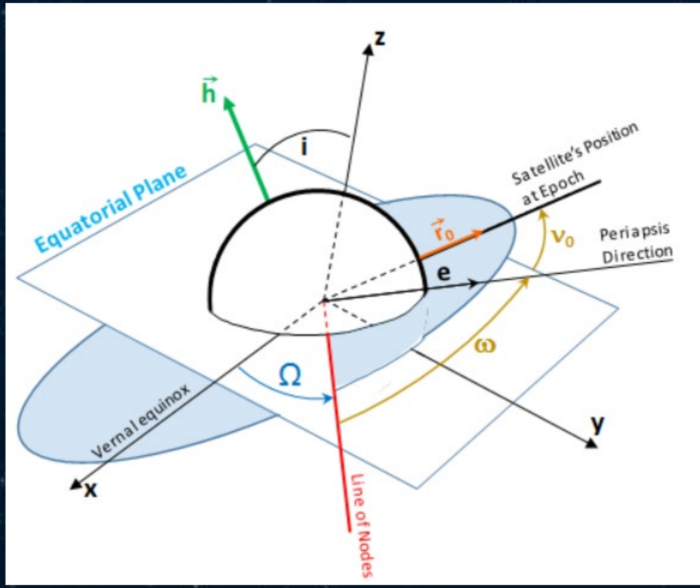
Quick Definitions

Relative Eccentricity Vector Δe (In-Plane Motion)

- The relative eccentricity vector is defined as

$$\Delta e = \begin{pmatrix} \Delta e_x \\ \Delta e_y \end{pmatrix} = \begin{pmatrix} \delta e * \cos \varphi \\ \delta e * \sin \varphi \end{pmatrix}$$

KEY MESSAGE: Defines radial and along-track oscillations.



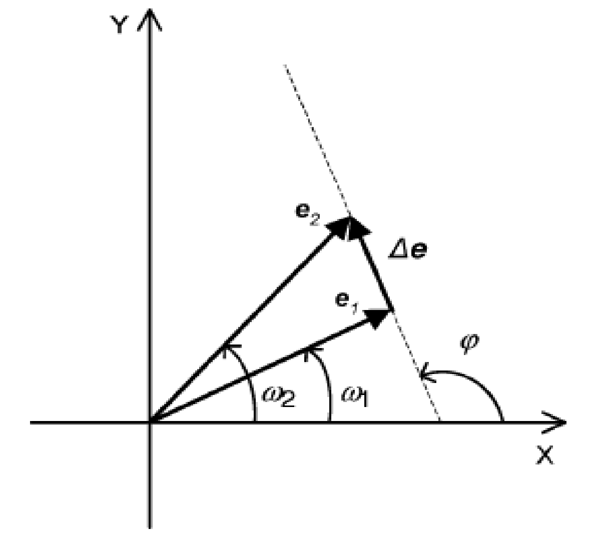


Fig. 3 Relative eccentricity vector definition.

D'Amico, Simone, and Oliver Montenbruck. "Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation." Journal of Guidance, Control, and Dynamics 29.3 (2006): 554-563.

Optimization for the XTI Harmony Mission Configuration

(in terms of the relative parameters Δe, Δi)

Quick Definitions

Relative Inclination Vector Δi (Out-of-Plane Motion)

$$\Delta \mathrm{i} = \begin{pmatrix} \Delta \mathrm{i}_x \\ \Delta \mathrm{i}_y \end{pmatrix} = \sin \delta i \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \sim \begin{pmatrix} \Delta i \\ \Delta \Omega \sin i \end{pmatrix}$$

KEY MESSAGE: Defines cross-track oscillations.

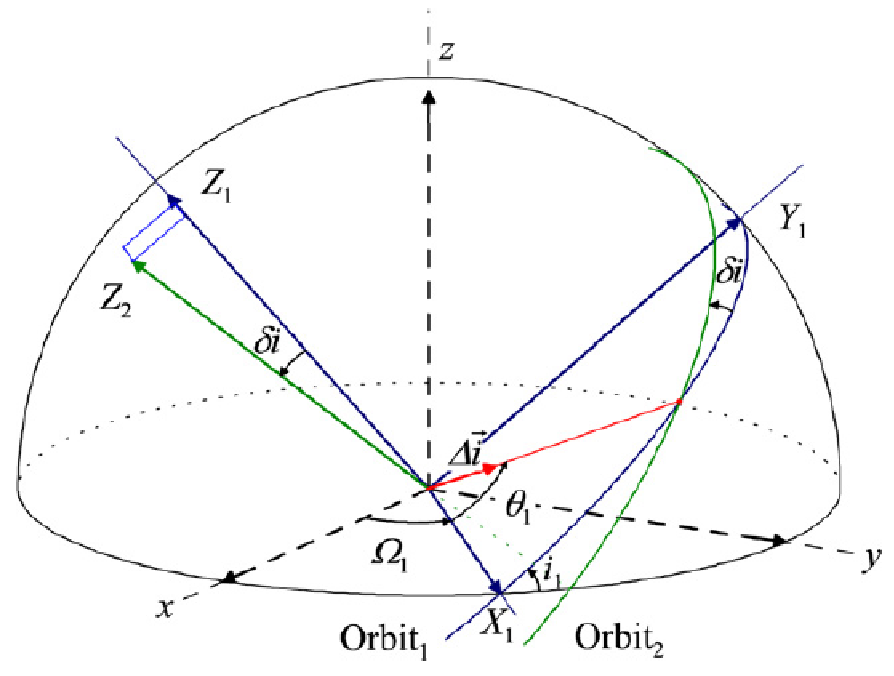


Fig. 1. Definition of the relative inclination vector.

D'Amico, Simone, and Oliver Montenbruck. "Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation." Journal of Guidance, Control, and Dynamics 29.3 (2006): 554-563.

Optimization for the XTI Harmony Mission Configuration

(in terms of the relative parameters Δe, Δi)

Quick Definitions

- $\varphi = \vartheta$ means Δe // Δi. When the condition of parallelism is verified, the minimum radial and cross track separations occur at different points of the orbit (safe condition).

Nominal Helix Formation ($\varphi = \vartheta$ = 90°)

- Only Y-components are non-zero

$\Delta e_x = \delta e * \cos\varphi$ and $\Delta e_y = \delta e * \sin\varphi$

$\Delta i_x = \delta i * \cos\theta$ and $\Delta i_y = \delta i * \sin\theta$

$\Delta i \sim \begin{pmatrix} 0 \\ \Delta\Omega \sin i \end{pmatrix}$

D'Amico, Simone, and Oliver Montenbruck. "Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation." Journal of Guidance, Control, and Dynamics 29.3 (2006): 554-563.

Optimization for the XTI Harmony Mission Configuration

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Nominal Helix Formation ($\varphi = \vartheta$ = 90°)

- Only Y-components are non-zero

$\Delta e_x = \delta e * \cancel{\cos} \varphi$ and $\Delta e_y = \delta e * \sin \varphi$

$\Delta i_x = \delta i * \cancel{\cos} \theta$ and $\Delta i_y = \delta i * \sin \theta$

$\Delta i \sim \binom{0}{\Delta\Omega \sin i}$

The helix-safe formation with $\varphi = \vartheta$ = 90° minimizes the secular drift induced by Earth's oblateness perturbations, providing a passively stable.

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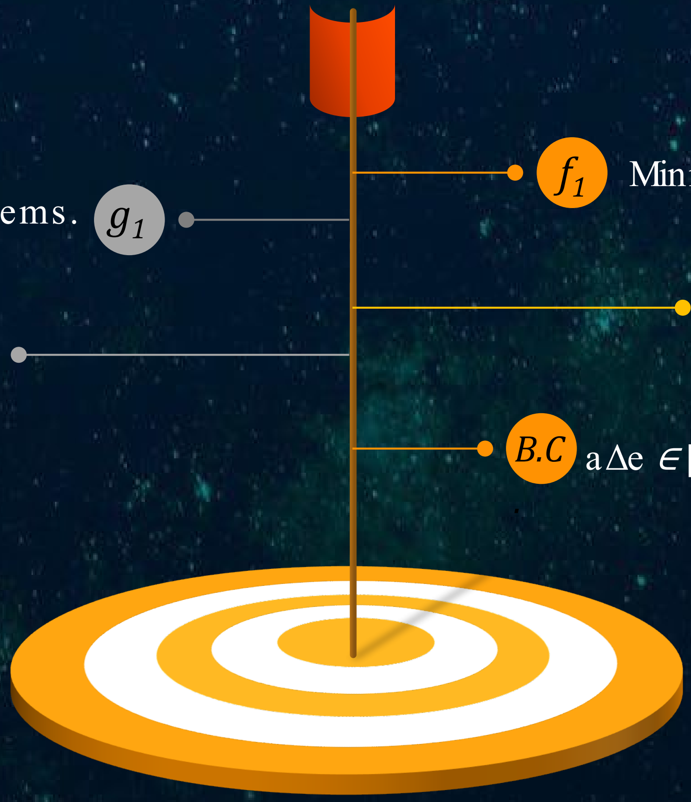
D'Amico, Simone, and Oliver Montenbruck. "Proximity operations of formation-flying spacecraft using an eccentricity/inclination vector separation." Journal of Guidance, Control, and Dynamics 29.3 (2006): 554-563.

Optimization for the XTI Harmony Mission Configuration

Python Optimization Algorithm to find $a\Delta e$ and $a\Delta\Omega$

$$\varphi = \vartheta = 90°$$



f_1 Minimize the HoA to increase the sensitivity.

g_1 HoA > 29 m to avoid PhU problems.

f_2 Minimize the temporal lag (critical for oceanographic applications).

g_2 the temporal lag $\in$ [-15, 15] ms.

B.C $a\Delta e \in [200, 750]$ and $a\Delta\Omega \in [900, 2100]$.

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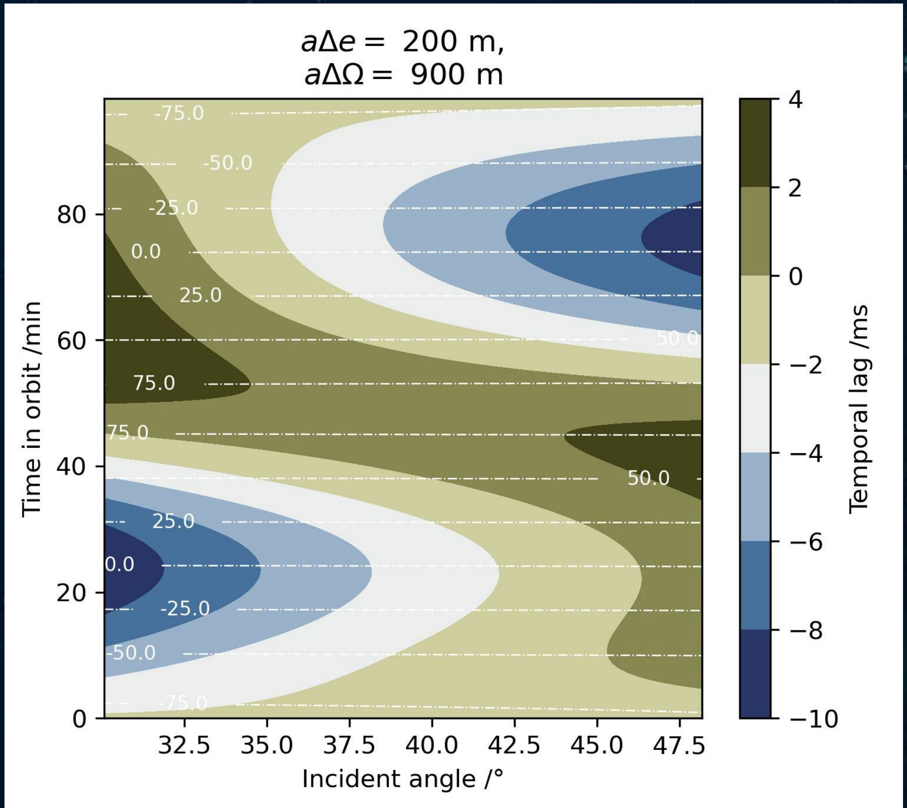
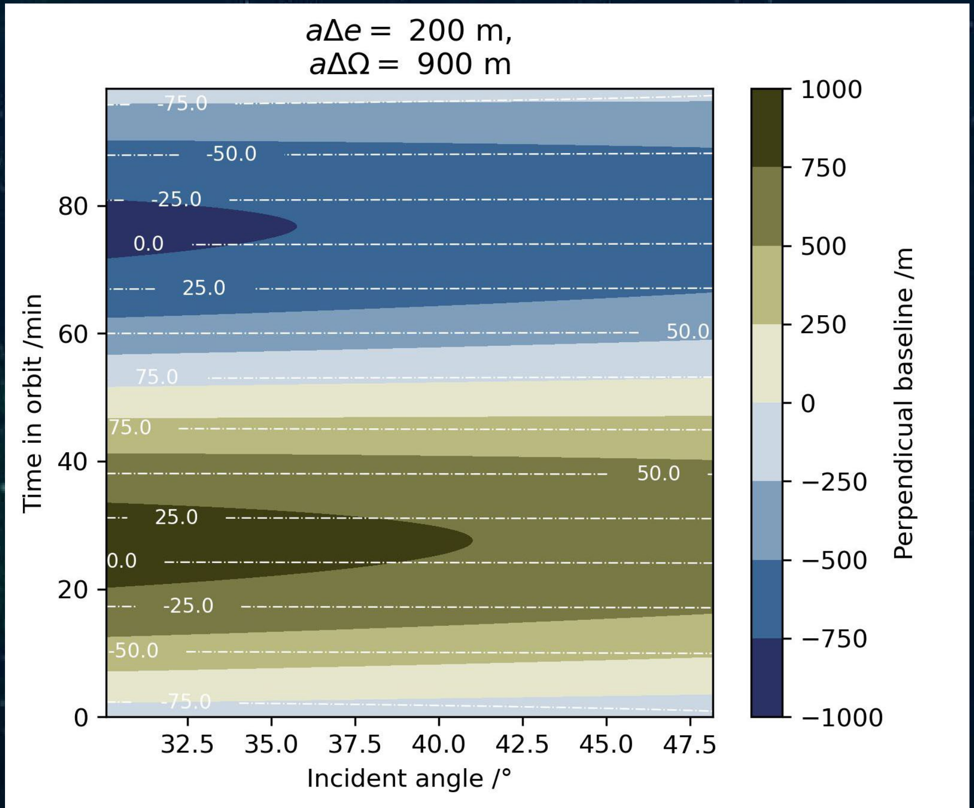
Preliminary Results

$\varphi = \vartheta = 90°$





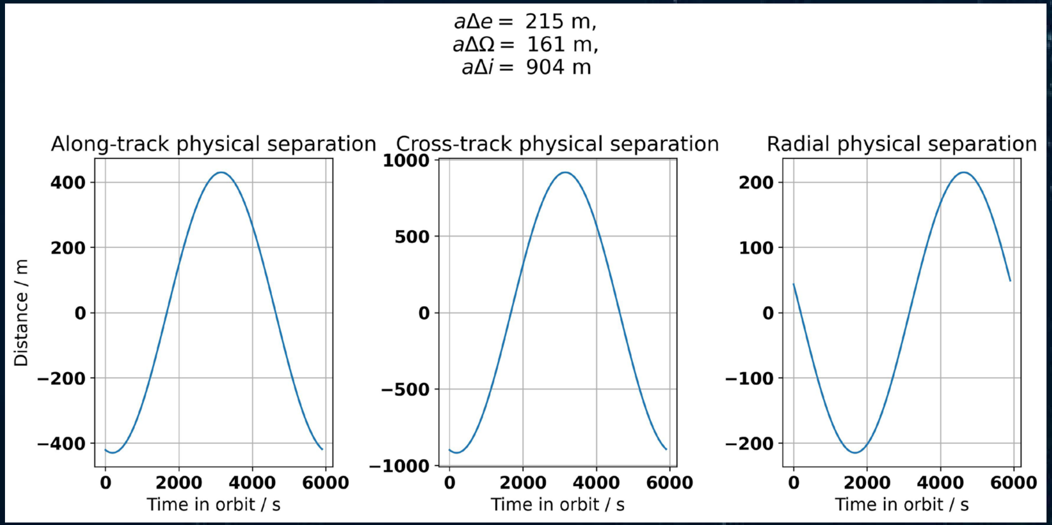
Min HoA ~ 30 m

A. Theodosiou, M. Kleinherenbrink and P. López-Dekker, "Wide-Swath Ocean Altimetry Using Multisatellite Single-Pass Interferometry," in IEEE Transactions on Geoscience and Remote Sensing, vol. 61, pp. 1-21, 2023, Art no. 5210721.

How to change where the minimum and maximum baseline values occur?

First Approach

- Choosing $\varphi = \vartheta = 10°$.



- Selecting $\vartheta \neq 90°$ introduces a difference in the orbital inclination between the two Harmony satellites.

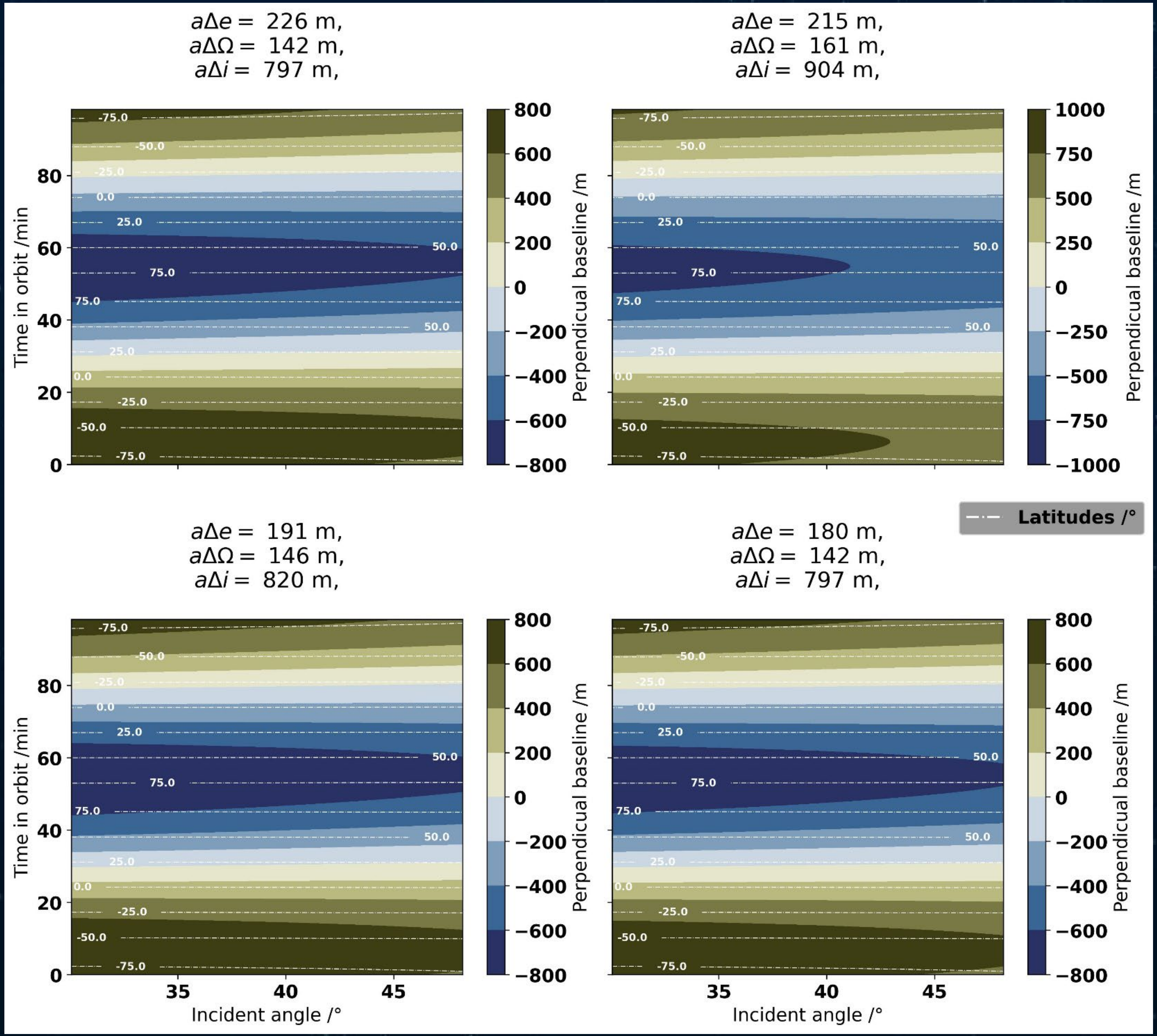
$$\Delta i \approx \left\{ \begin{array}{c} \Delta i \\ \Delta\Omega \sin i \end{array} \right\} \Longrightarrow \theta = \tan^{-1} \frac{\Delta\Omega \sin i}{\Delta i}$$

Second Approach

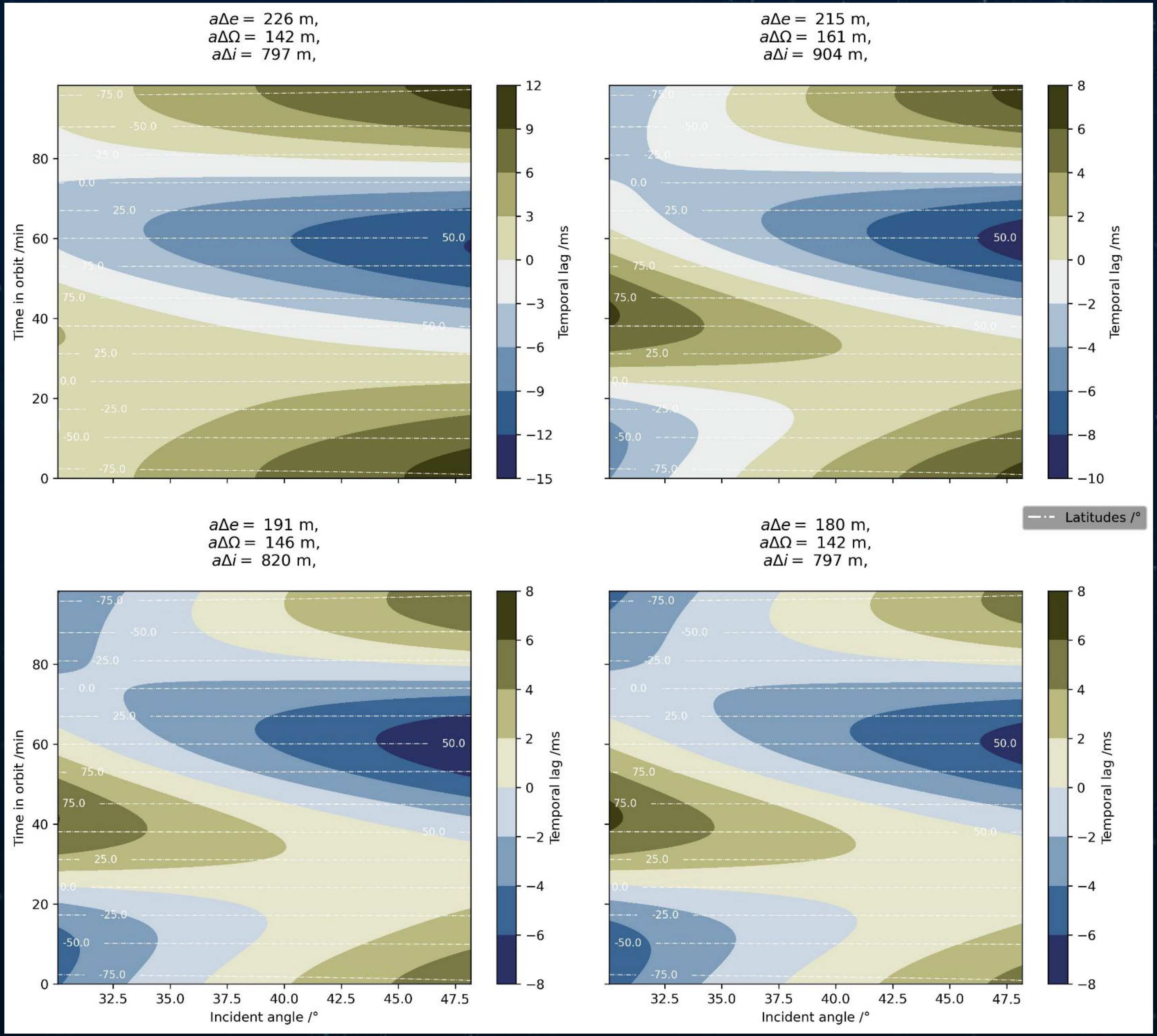
- Keeping $\varphi = \vartheta = 90°$ (passively stable configuration).

- Finding an optimal configuration for polar latitudes.

First Approach: Optimal configurations for $\varphi = \vartheta = 10°$



First Approach: Optimal configurations for $\varphi = \vartheta = 10°$



First Approach: Optimal configurations for $\boldsymbol{\varphi} = \boldsymbol{\vartheta} = 10°$

Effects of Introducing Δi ≠ 0

- Due to the **J2 perturbation**, the component Δi_Y increases linearly over time

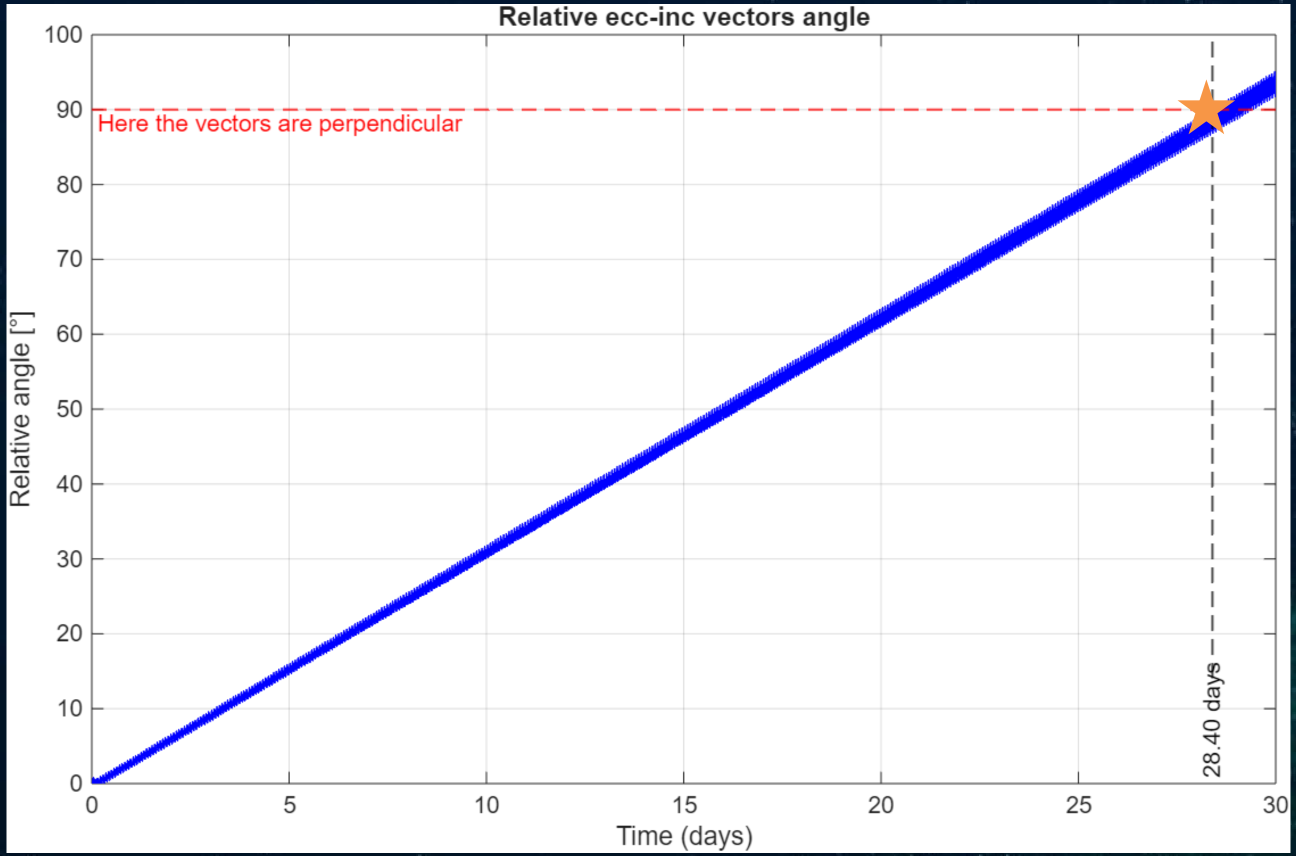
$$\Delta\bar{i} = \begin{Bmatrix} \Delta\bar{i}_X \\ \Delta\bar{i}_Y \end{Bmatrix} = \begin{Bmatrix} \Delta i_X \\ \Delta i_Y + \mathrm{d}(\Delta i_Y)/\mathrm{d}t \cdot t \end{Bmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta i_Y \approx -\frac{3\pi}{T} J_2 \frac{R_\oplus^2}{a^2} \sin^2(i) \cdot \Delta i$$

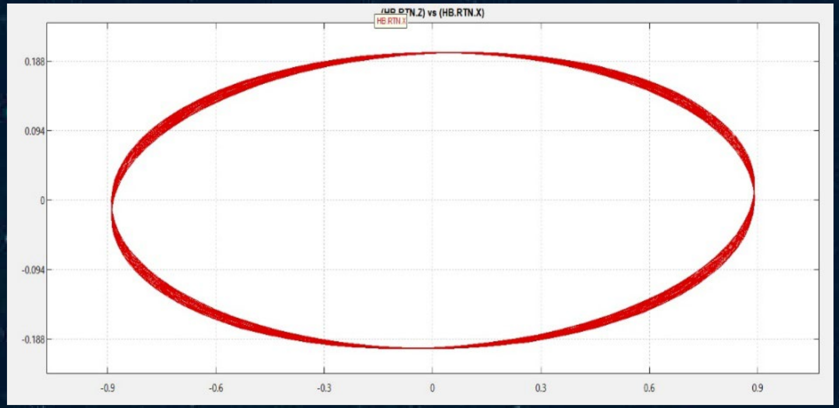
- Result: Angle between the relative vectors increases over time → Critical 90° alignment.

- Consequently, periodic maneuvers will be needed to control this drift.

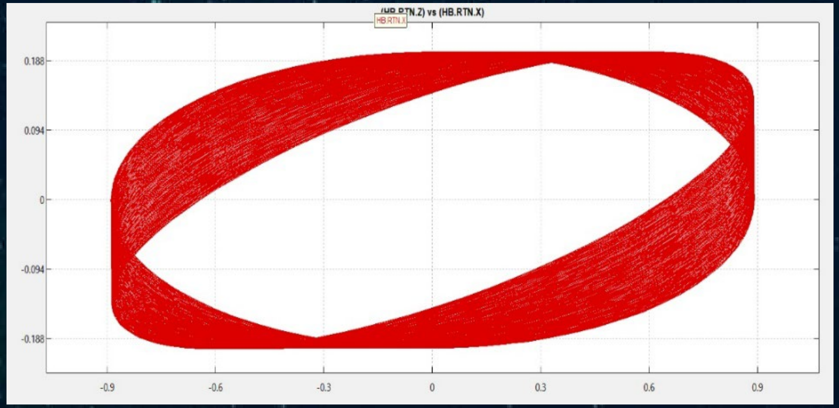
$\varphi = \vartheta = 90°$ $a\Delta e$ = 200 m and $a\Delta\Omega$ = 900 m



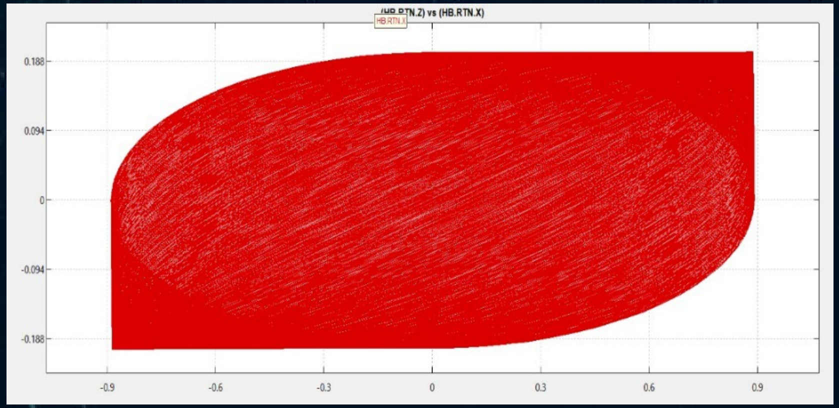
Δe and Δi become perpendicular after ~ 28 days → delayed risk



2 days



14 days



28 days

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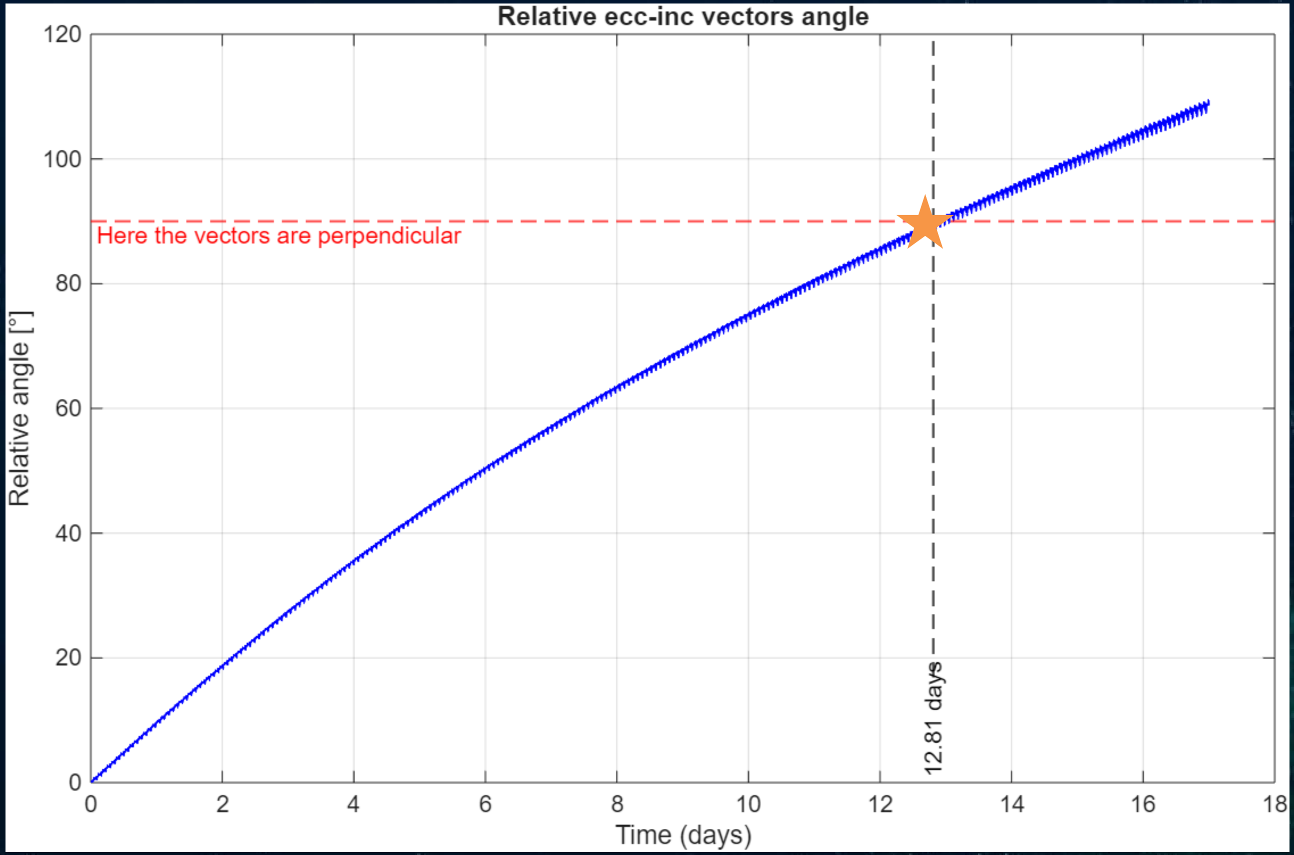
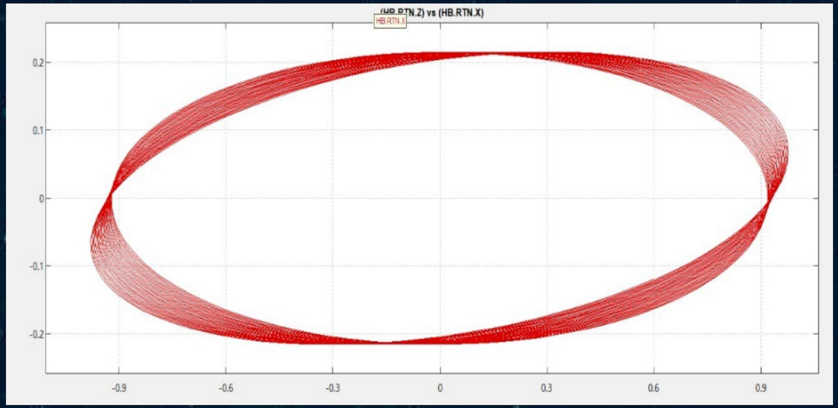
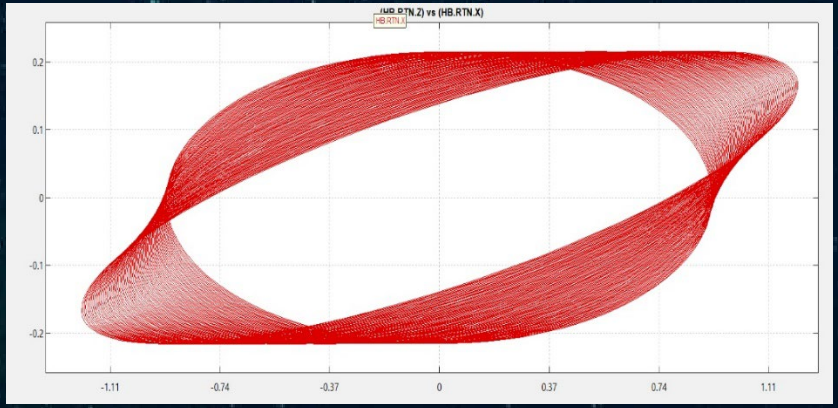
$\varphi = \vartheta = 10°$ $a\Delta e = 215$ m and $a\Delta\Omega = 161$m



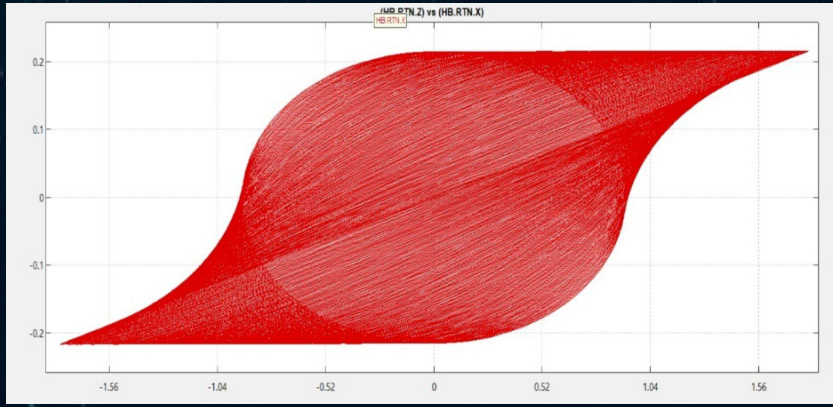
Δe and Δi become perpendicular after ~ 13 days → early risk



2 days



6 days



13 days

Second Approach

Python Optimization Algorithm to find aΔe and a$\Delta\Omega$

(only for the poles)

$$\varphi = \vartheta = 90°$$

- f_1: Sensitivity at the poles > Sensitivity at the equator.
- f_2: Minimize the HoA at the poles to increase the sensitivity.
- g_2: HoA > 29 m to avoid PhU problems.
- B.C: aΔe and a$\Delta\Omega$ at least 100 meters (for safety reasons).

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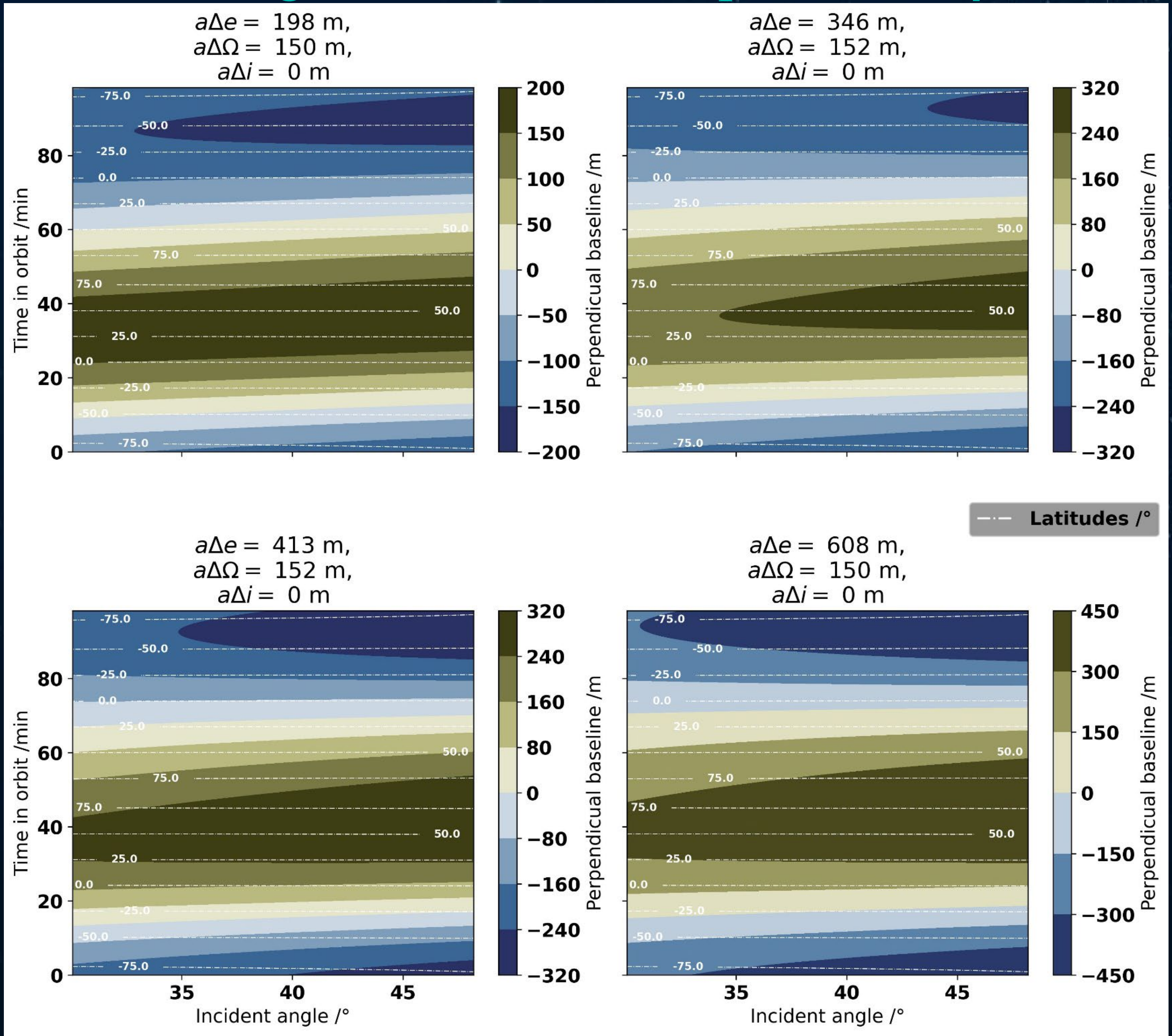
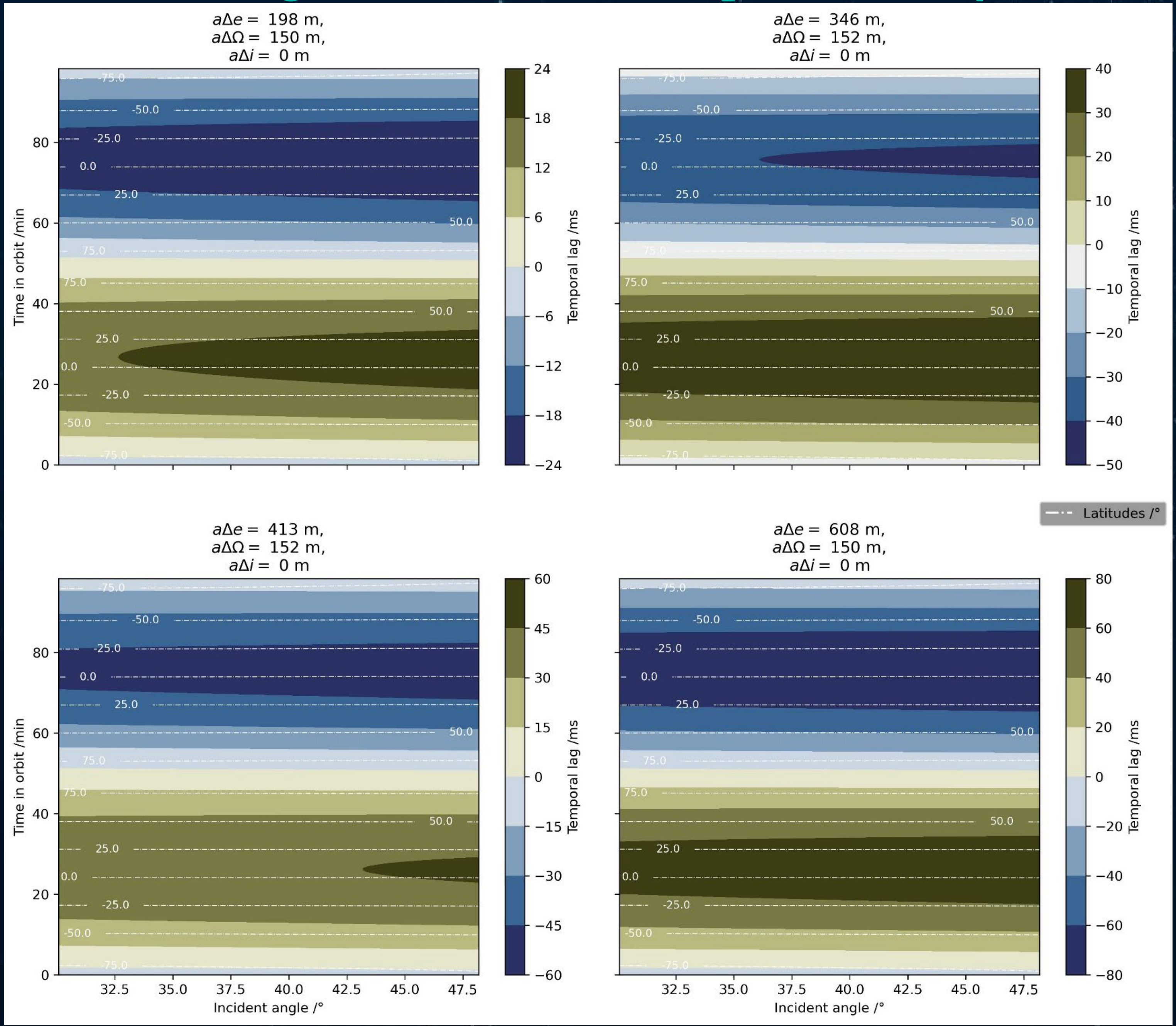
Optimal configurations for the poles for $\varphi = \vartheta = 90°$



Optimal configurations for the poles for $\varphi = \vartheta = 90°$



How to change where the minimum and maximum baseline values occur?

First Approach

- Choosing $\varphi = \vartheta = 10°$.
  - Higher values of perpendicular baseline (up to 1000 m) and sensitivity.
  - Low temporal lag values everywhere.
  - ΔV for maintaining the formation increases by two orders of magnitude.

Second Approach

- Keeping $\varphi = \vartheta = 90°$ (passively stable configuration).
  - Baseline geometry shifted to maximize sensitivity at the poles (zero now at the equator).
  - Max perpendicular baseline values are moderate (up to 400 m).
  - Temporal lag kept low, but only at polar latitudes.
  - ΔV budget respected both for maintaining formation and switching between configurations.

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