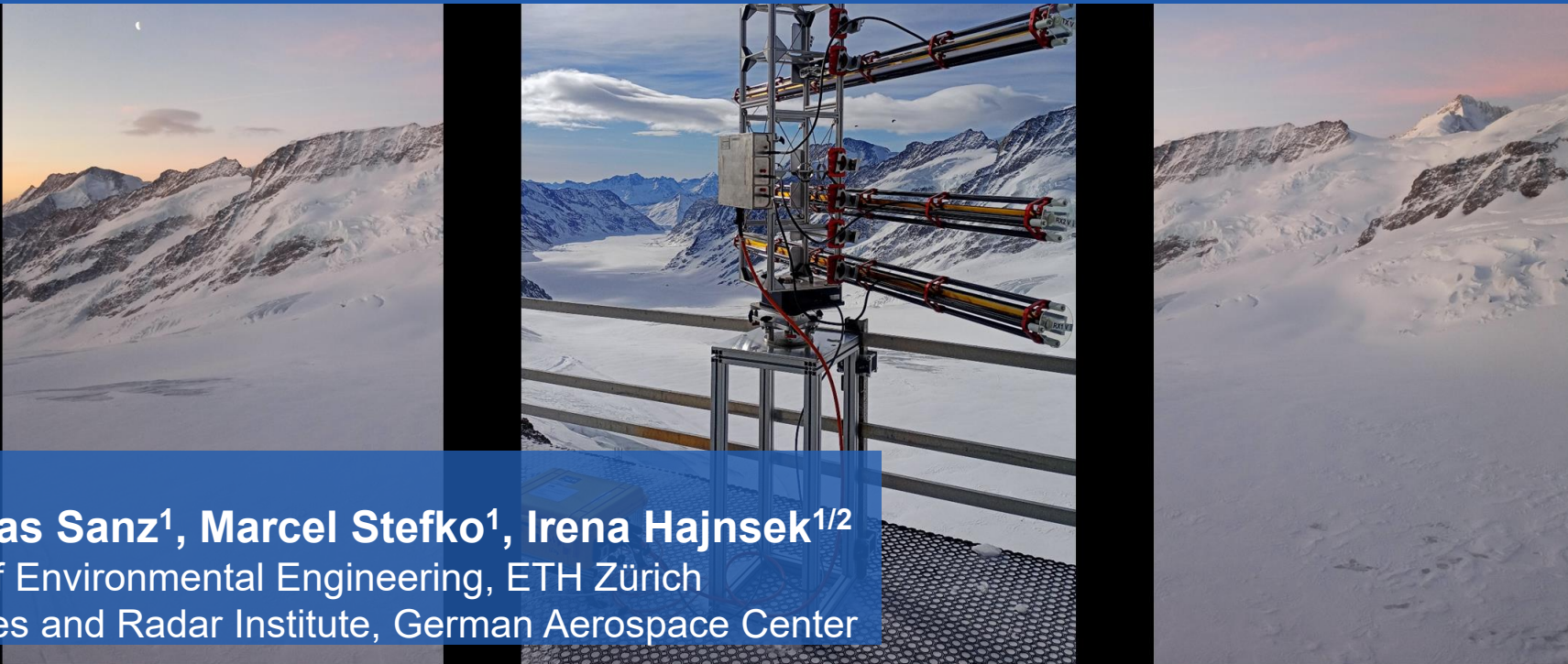


# DEM-assisted 3D reconstruction of Aletsch glacier displacements using monostatic and bistatic differential interferometry



**Esther Mas Sanz<sup>1</sup>, Marcel Stefko<sup>1</sup>, Irena Hajnsek<sup>1/2</sup>**

<sup>1</sup>Institute of Environmental Engineering, ETH Zürich

<sup>2</sup>Microwaves and Radar Institute, German Aerospace Center

# Introduction to KAPRI

*Ku-Band Advanced Polarimetric Radar Interferometer*

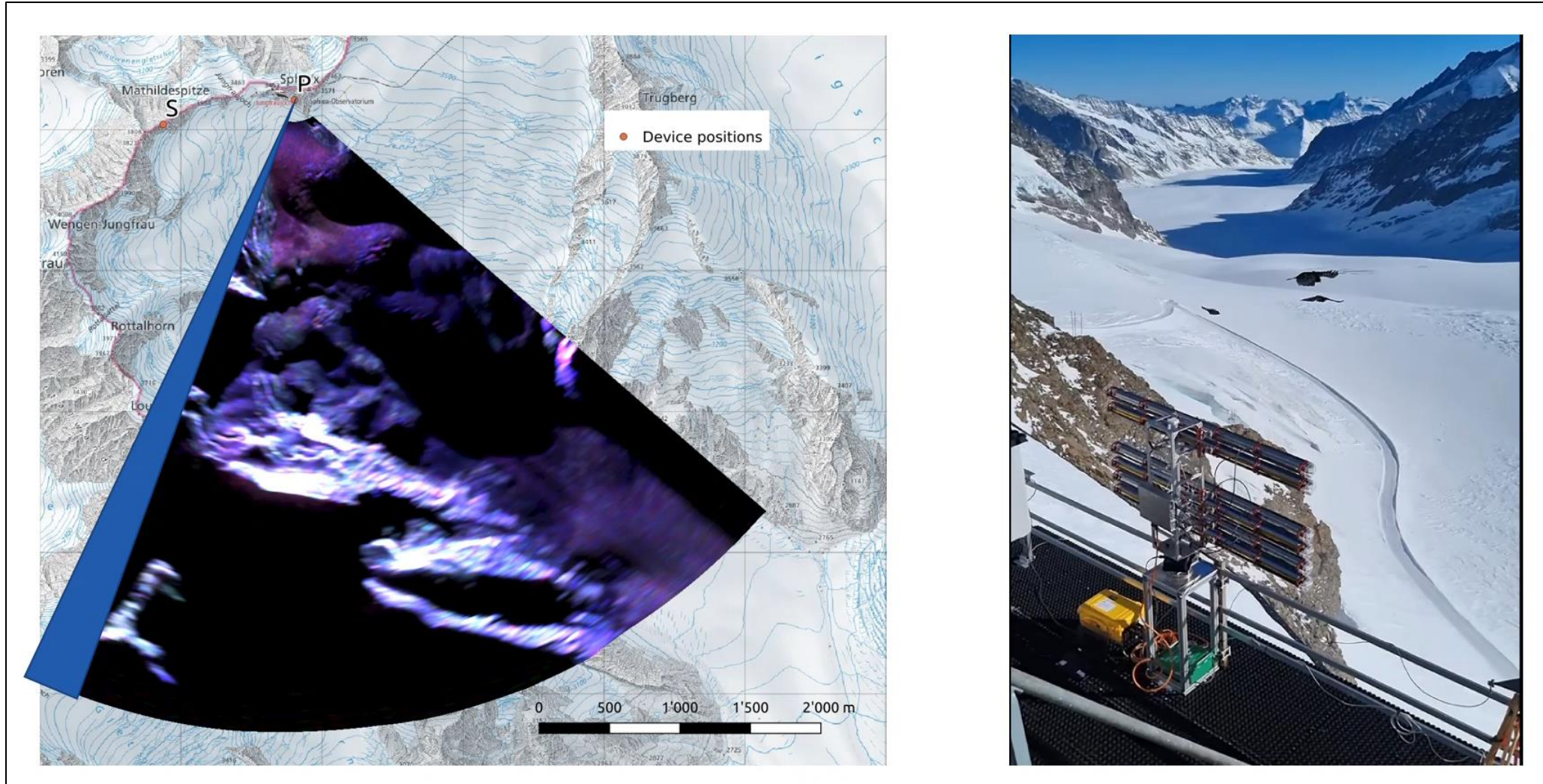
*GAMMA REMOTE SENSING*



Specifications	
Frequency	17.2 GHz
Type	FMCW
Bandwidth	200 MHz
Polarization TX/RX	V, H
Range	50m – 10km
Operation modes	Monostatic, Bistatic
Other characteristics	Real Aperture

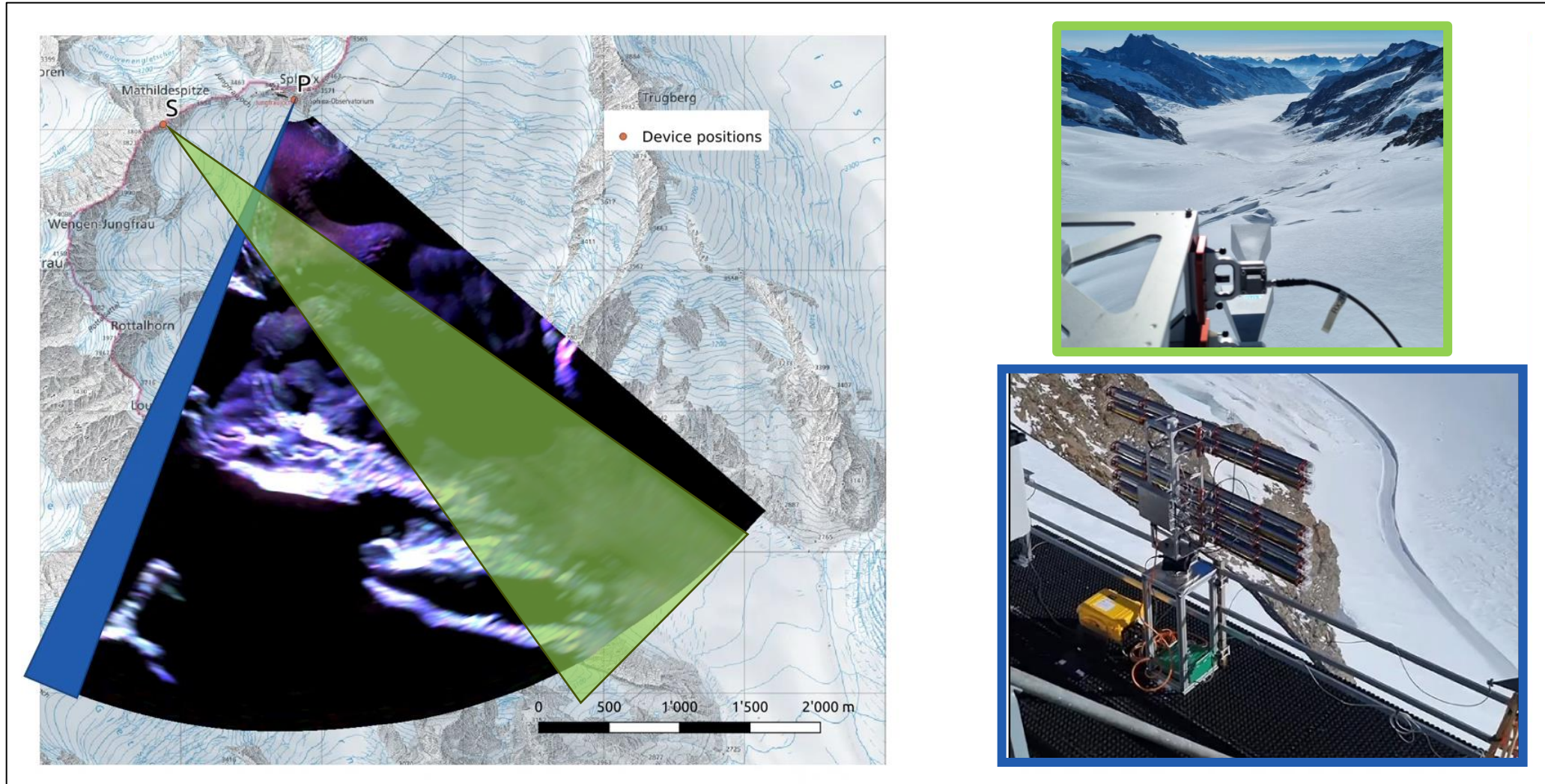


# Introduction to KAPRI: Monostatic KAPRI operation





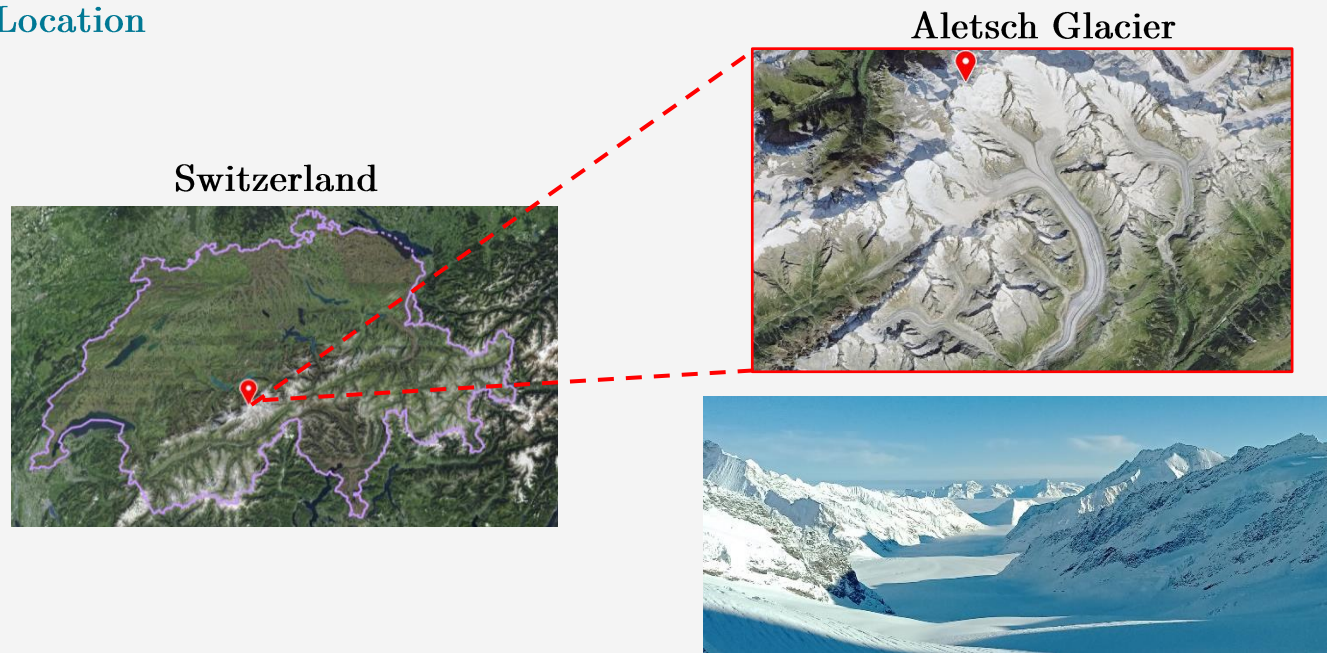
# Introduction to KAPRI: Bistatic KAPRI operation





# Description of the Dataset

## Location



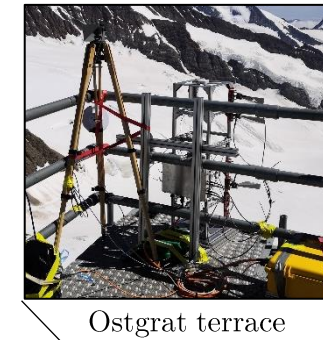
## Specifications

Dataset Specs	
Date	2 <sup>nd</sup> March 2022
Polarisation	Full-Pol (VV)
Repeat time	1.5 min
Time series	1h (9AM-10AM)
Min Range	200m
Max Range	4km

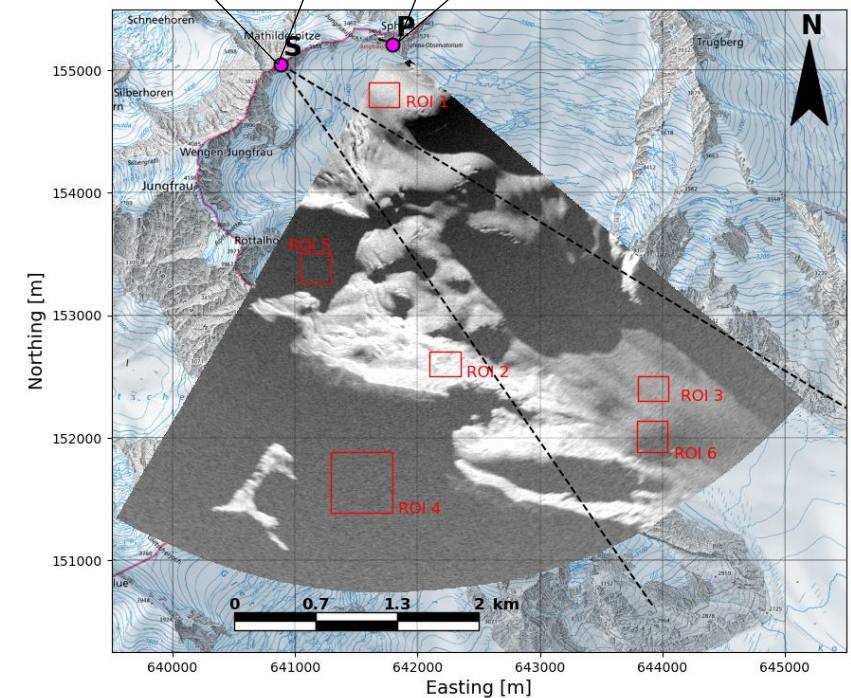
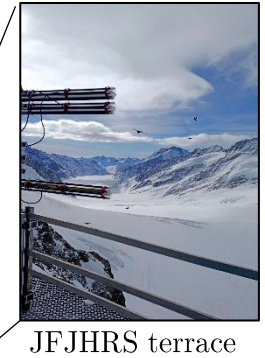
## Regions of Interest (ROIs)

ROI Number	Description
1	Near Range
2	Mid Range
3	Far Range
4	Noise far range
5	Noise near range
6	Noise far range (only bistatic)

## Secondary



## Primary



# Displacement Calculation: Monostatic Case

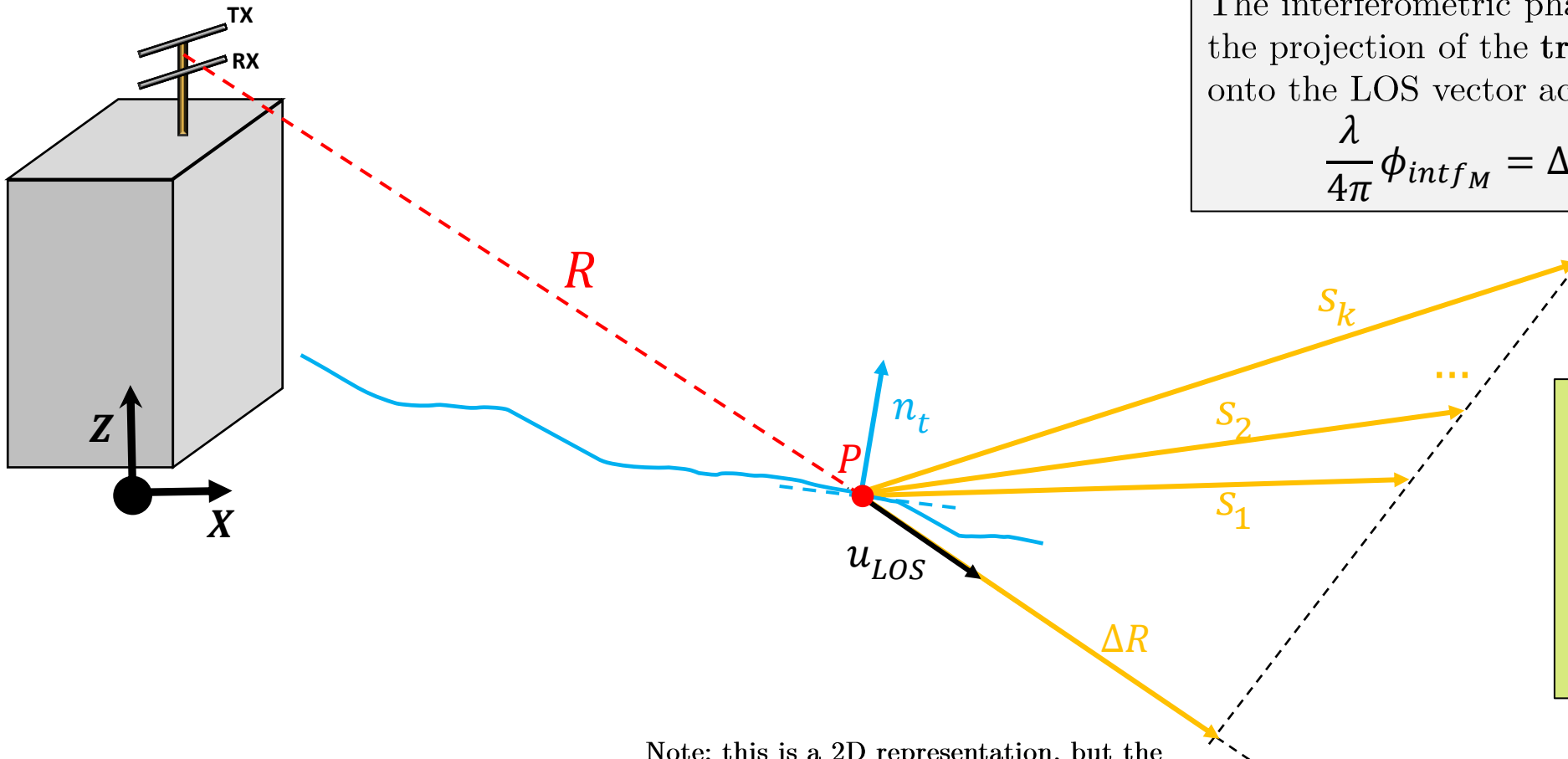
$$R_{TX} = R_{RX}$$

The TX leg is the same as the RX leg!

$$\text{Interferogram [mono]: } \phi_{intf_M} = \frac{4\pi}{\lambda} \Delta R$$

The interferometric phase is proportional to the projection of the **true displacements** onto the LOS vector according to:

$$\frac{\lambda}{4\pi} \phi_{intf_M} = \Delta R = \vec{s}_k \cdot \vec{u}_{LOS}$$



Imposing that displacement vector direction corresponds to the maximum gradient  $\nabla f$

$$\vec{s} = \nabla f \cdot |\vec{s}|$$

Then, there system is completely defined

$$\vec{s} \cdot \vec{u}_{LOS} = |\vec{s}| \nabla f \cdot \vec{u}_{LOS} = \Delta R$$

Note: this is a 2D representation, but the problem is 3D. In a general case  $\vec{u}_{LOS}$ ,  $\vec{n}_t$ ,  $\vec{s}$  are not contained in the same plane

# Displacement Calculation: Bistatic Case

The TX leg and RX leg have different look vectors!

$$\text{Interferogram [bi]: } \phi_{intf_B} = \frac{2\pi}{\lambda} (\Delta R_{TX} + \Delta R_{RX})$$

Where  $\frac{2\pi}{\lambda} \Delta R_{TX}$  is equivalent to half of the monostatic interferometric phase  $\frac{2\pi}{\lambda} \Delta R_{TX} = \frac{\phi_{interf_M}}{2}$

Each interferometric phase is proportional to the projection of the **true displacements** onto the respective LOS vector:

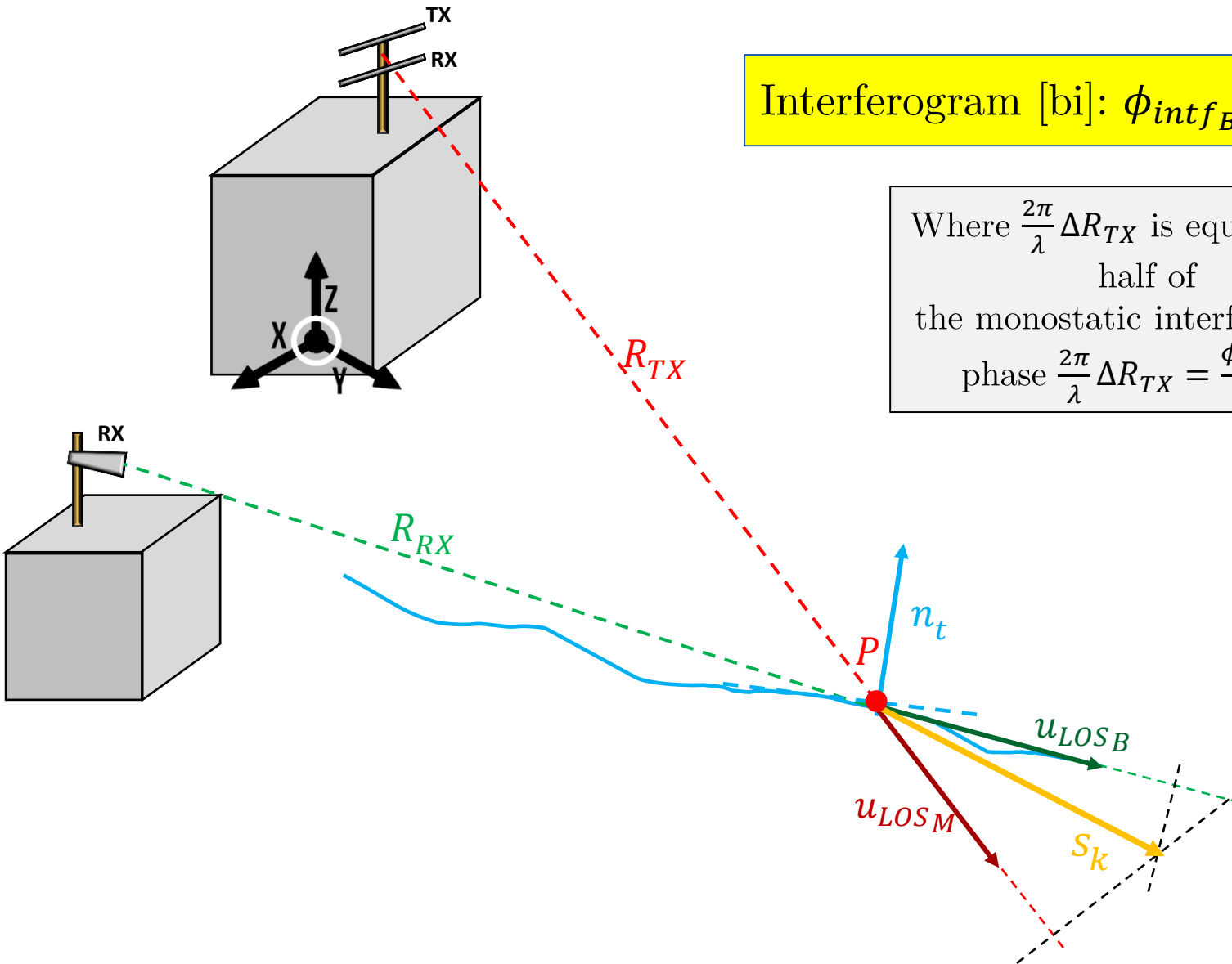
$$\frac{\lambda}{4\pi} \phi_{intf_M} = \Delta R_{TX} = \vec{s}_k \cdot \vec{u}_{LOS_M}$$

$$\frac{\lambda}{2\pi} \phi_{intf_{RX\ leg}} = \Delta R_{RX} = \vec{s}_k \cdot \vec{u}_{LOS_B}$$

Imposing that displacement vector is contained in the plane tangential to the surface. In other words, that the dot-product between the normal of the surface and the true displacements is 0:

$$\vec{s} \cdot \vec{n}_t = 0$$

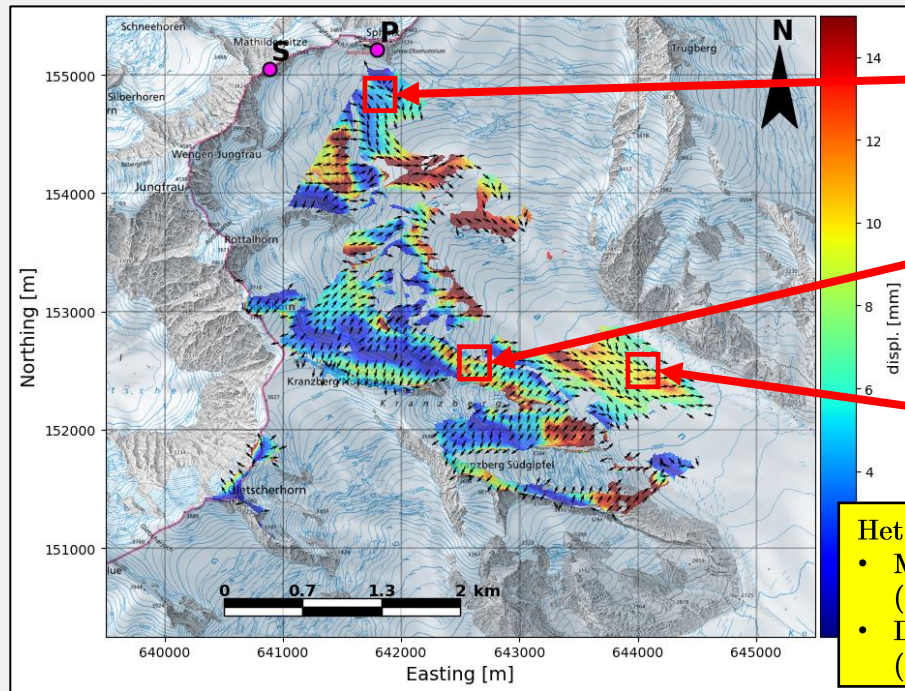
Together with LOS equations, it forms an independent system with one solution





# Displacements results: Displacement Vector Field

Monostatic

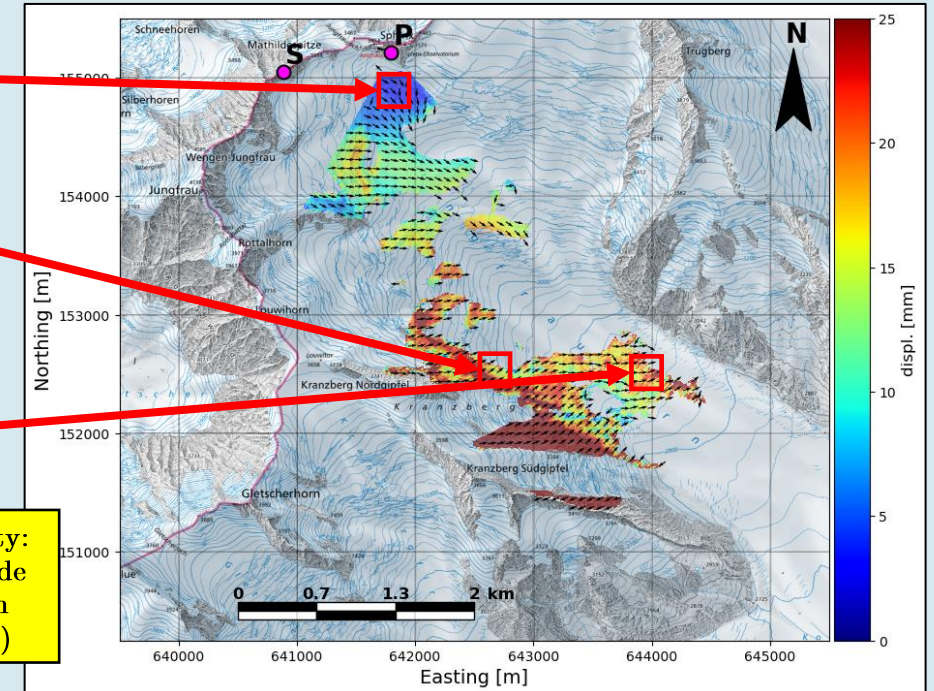


ROI 1  
ROI 2  
ROI 3  
displacement map

Heterogeneity:  
• Magnitude (across scene)  
• Direction (steep regions)

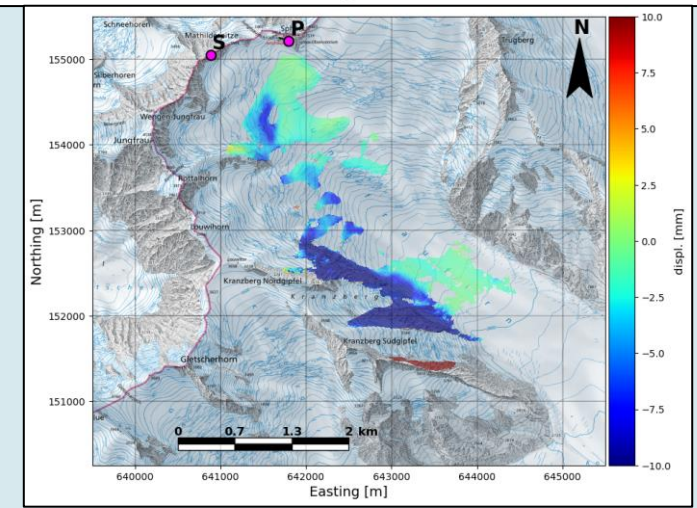
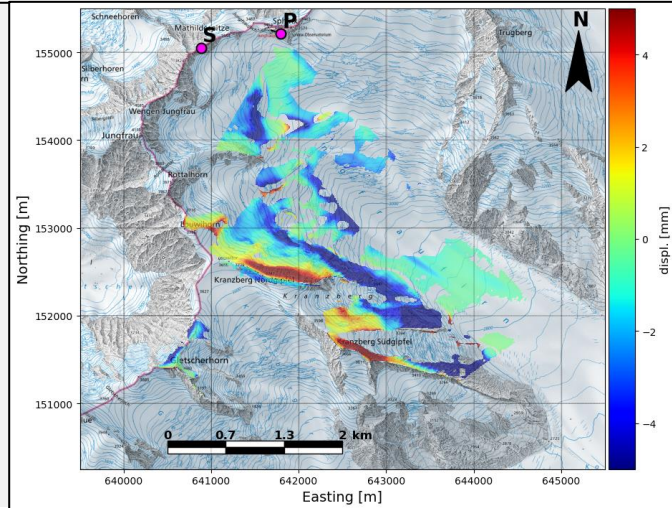
Homogeneity:  
• Magnitude (near range)  
• Direction

Bistatic



Z-direction

Glacier region  
 $\Delta_z \approx 0$  m  
Steep regions  
( $\Delta_z \neq 0$  m)

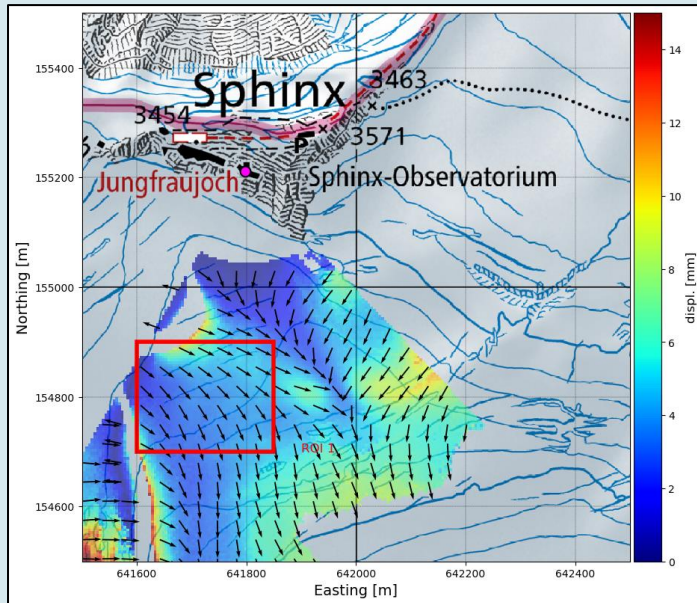




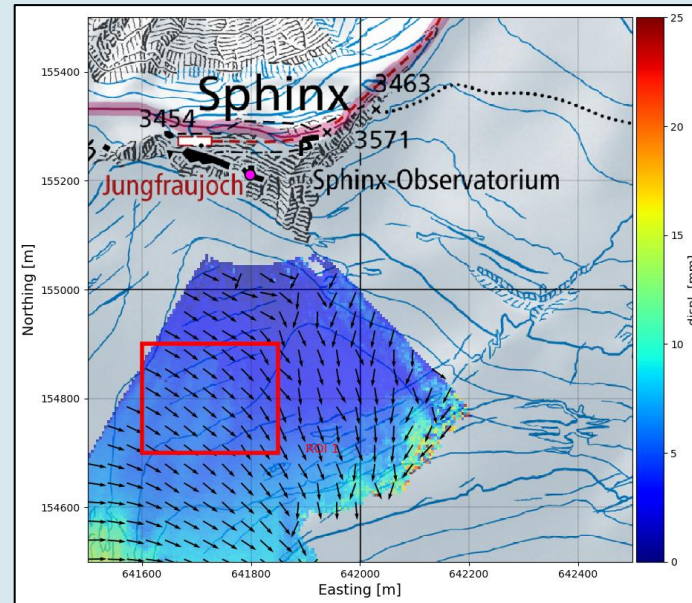
# Displacements results: Bearing, Elevation and Magnitude

ROI 1 Zoom-In

Monostatic

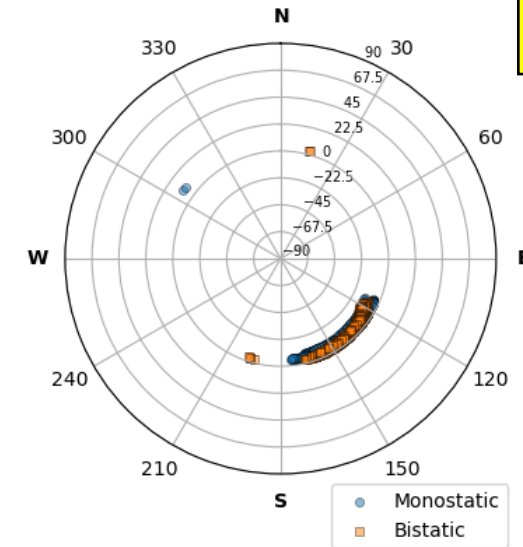


Bistatic

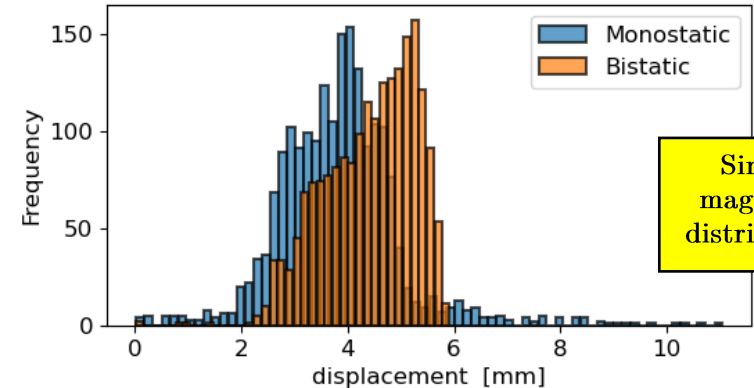


Estimation of very similar displacement field by the two methods

ROI 1 (near range glacier)



Same direction (3D)

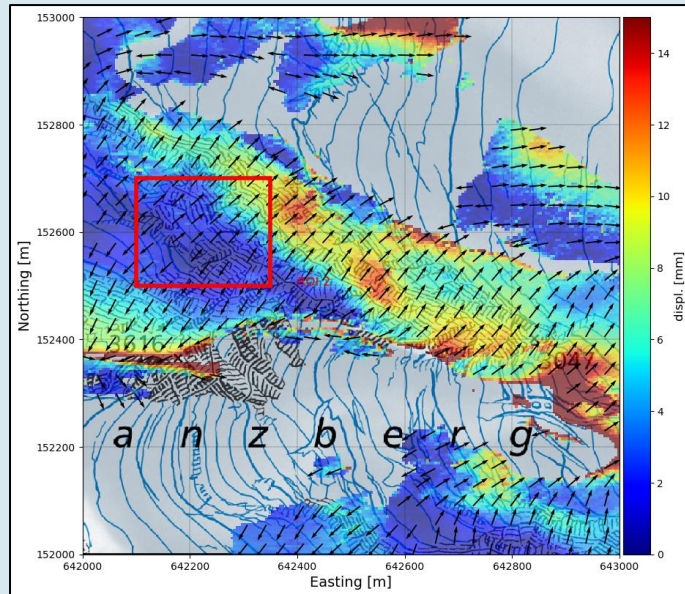


Similar magnitude distributions

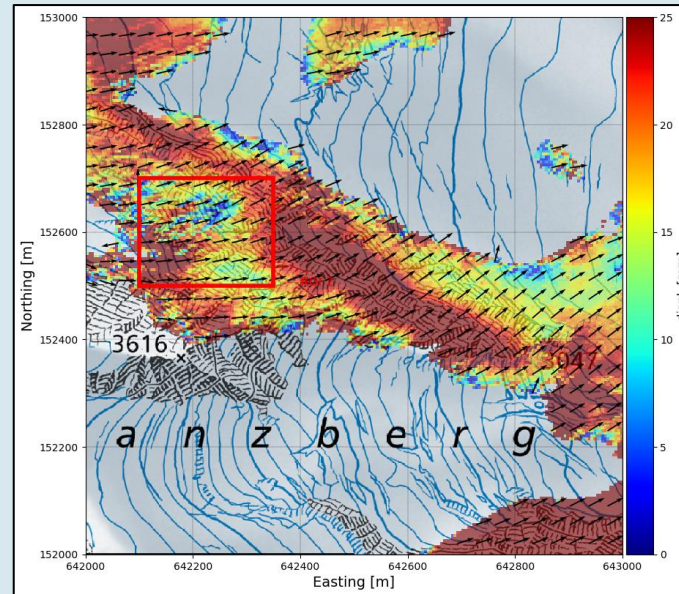
# Displacements results: Bearing, Elevation and Magnitude

ROI 2 Zoom-In

Monostatic

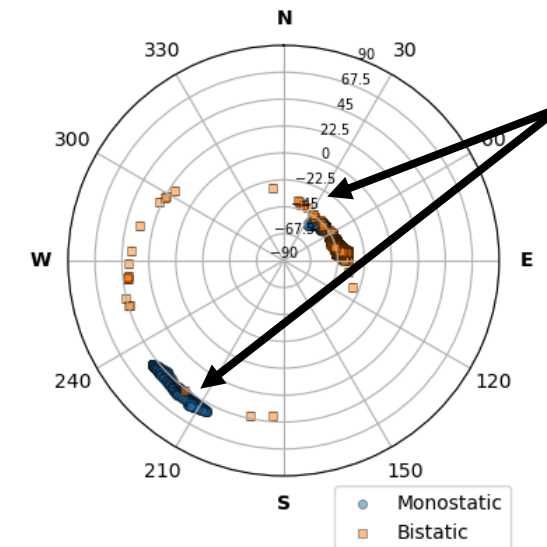


Bistatic

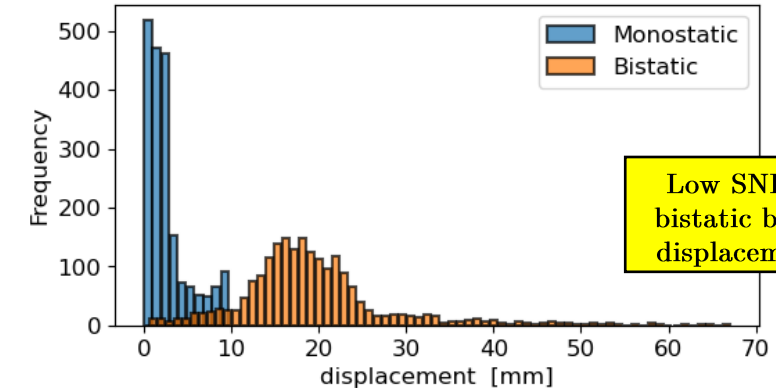


Unfavorable geometries in high steep regions + poor SNR (bistatic) lead to different displacement maps

ROI 2 (mid range, steep region)



- Monostatic: two data clusters in elevation ( $\pm \nabla f$ )
- Bistatic: bearing high std dev



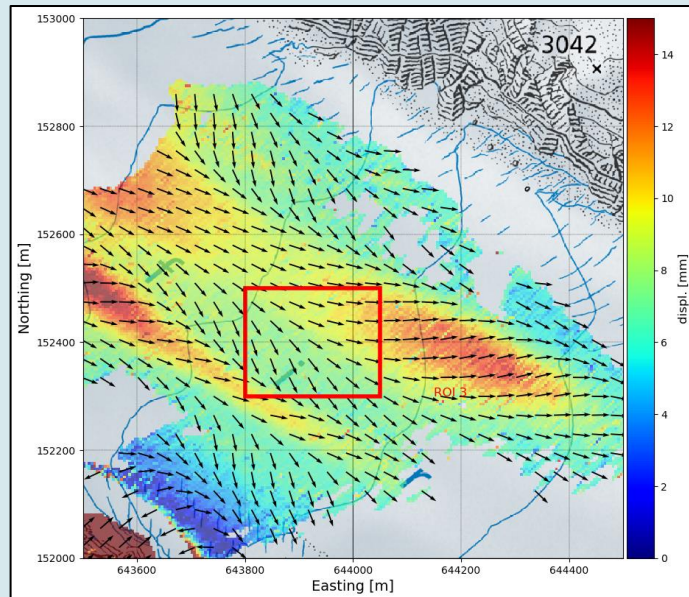
Low SNR in bistatic biases displacements



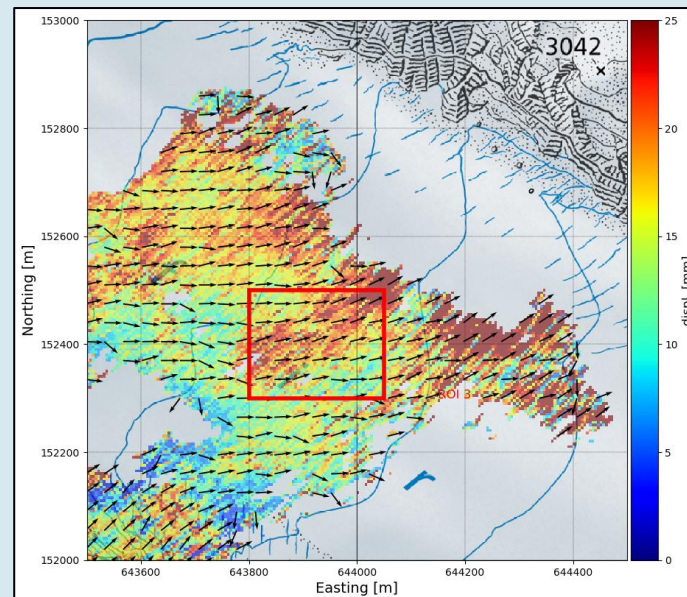
# Displacements results: Bearing, Elevation and Magnitude

ROI 3 Zoom-In

Monostatic



Bistatic



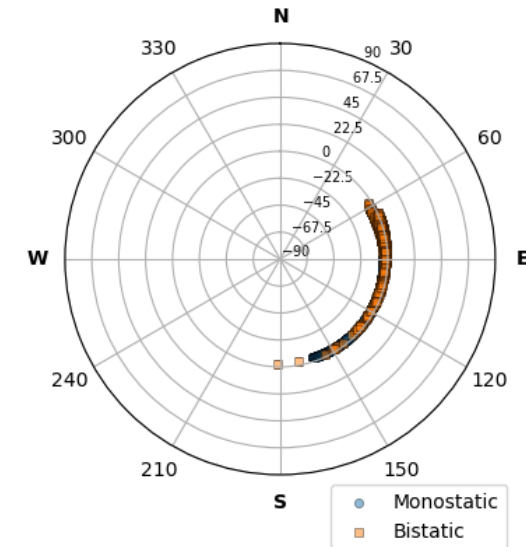
Monostatic:

- $SNR \geq 10dB$  in the far range
- Displacement direction fixed. Only magnitude affected

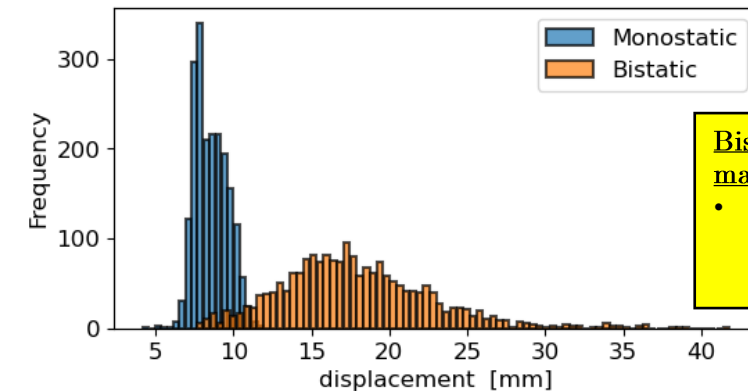
Bistatic:

- $SNR \leq 10dB$  in the far range
- Impact on magnitude and direction

ROI 3 (far range glacier)



Higher std dev on bearing (only DoF) → SNR lower



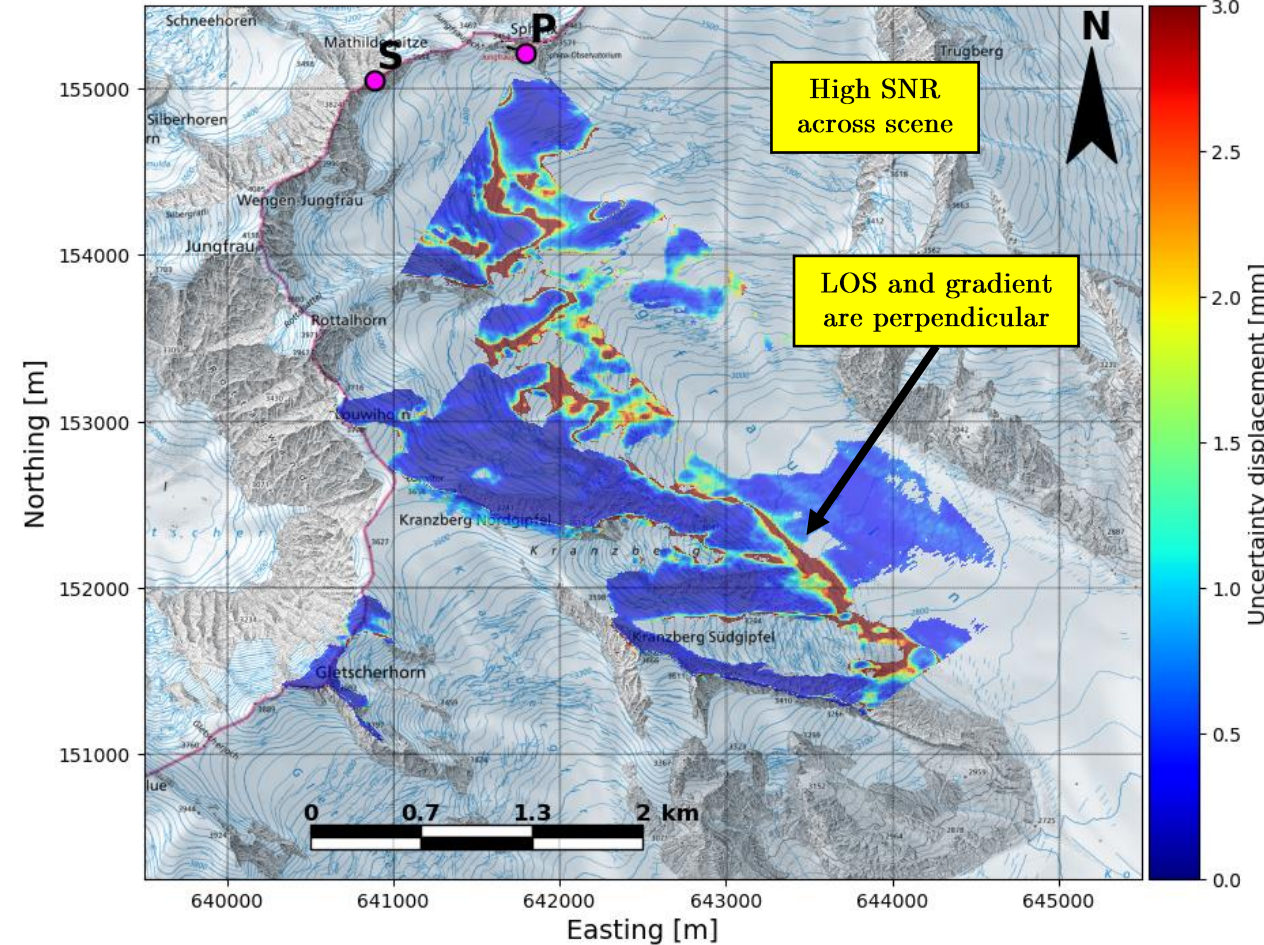
Bistatic magnitude:

- flattened distribution → Higher std dev



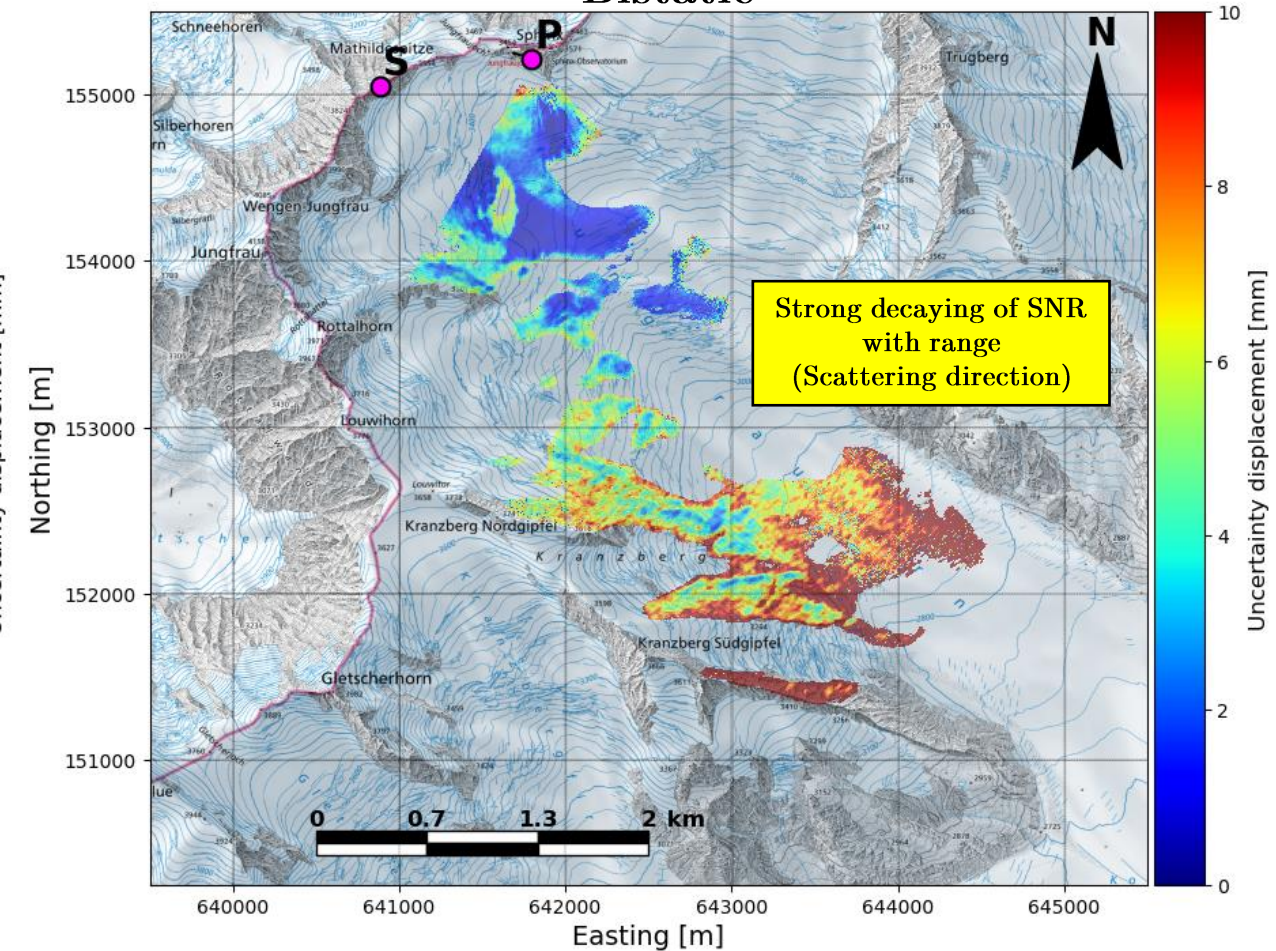
# Displacements results: Displacement Uncertainty

Monostatic



$$\sigma_{||s||} = \frac{\lambda}{4\pi(\hat{g} \cdot \vec{u}_m)} \sigma_{\phi_m} \quad \sigma_{\phi} = \frac{1}{\sqrt{SNR}}$$

Bistatic



$$\sigma_{||s||} = \sqrt{\left(\frac{\partial ||\vec{s}||}{\partial \phi_m}\right)^2 \sigma_{\phi_m}^2 + \left(\frac{\partial ||\vec{s}||}{\partial \phi_s}\right)^2 \sigma_{\phi_s}^2}$$

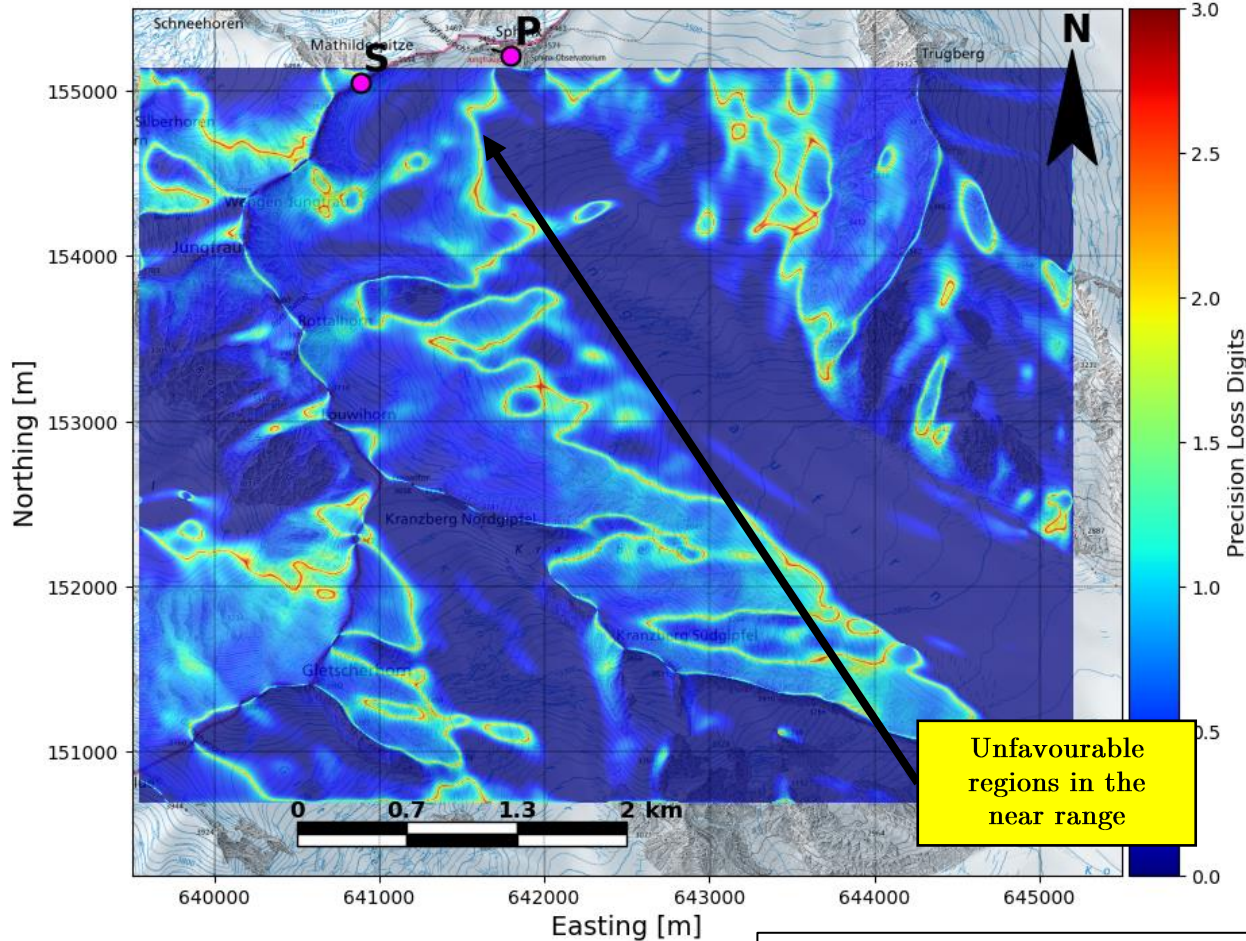


# Displacements results: Digit Precision Loss

(factor for error propagation)

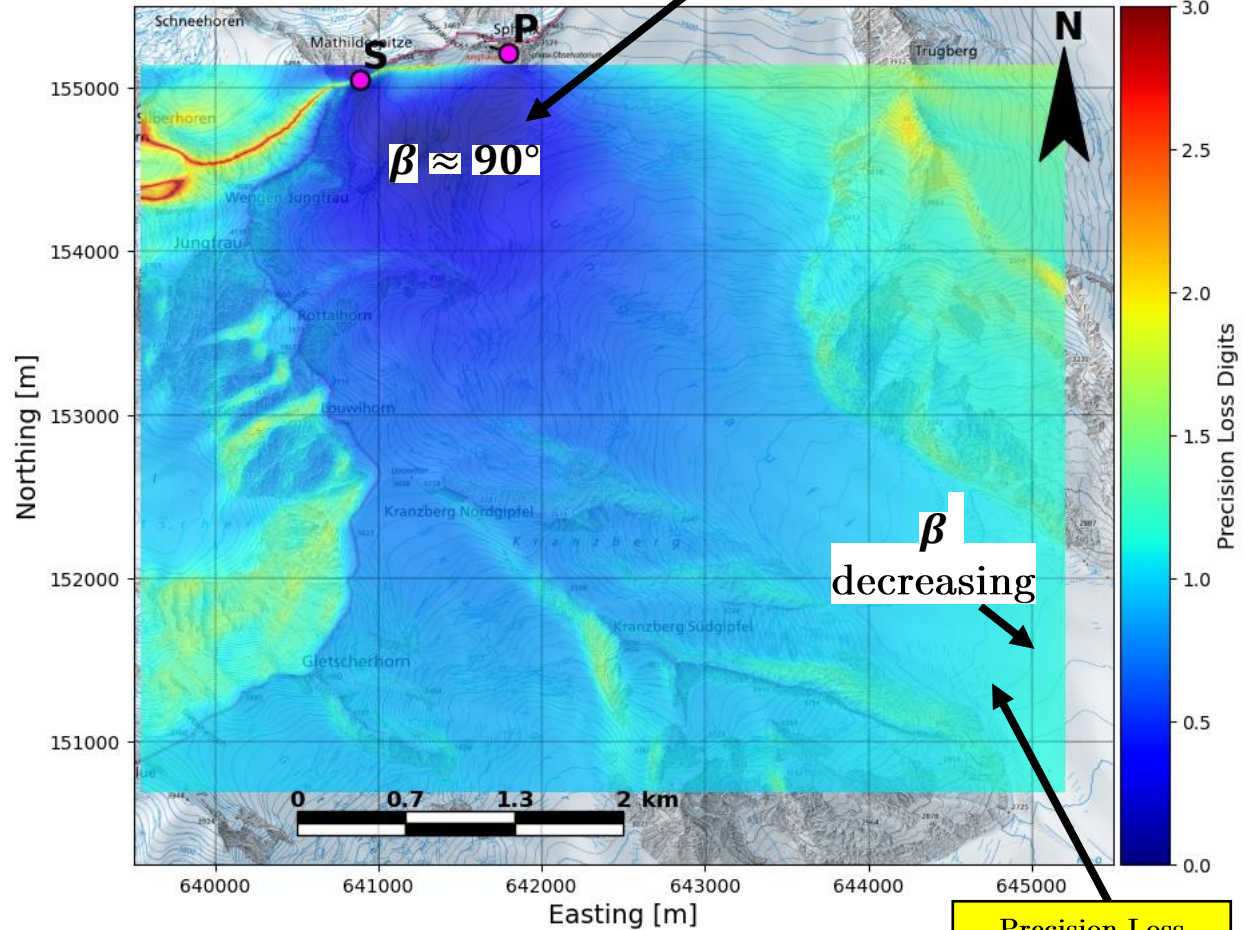
Monostatic

$$\kappa_m = \|\gamma \tan \gamma\|$$



Bistatic

$$\kappa(A) = \|A\| \cdot \|A^{-1}\|$$

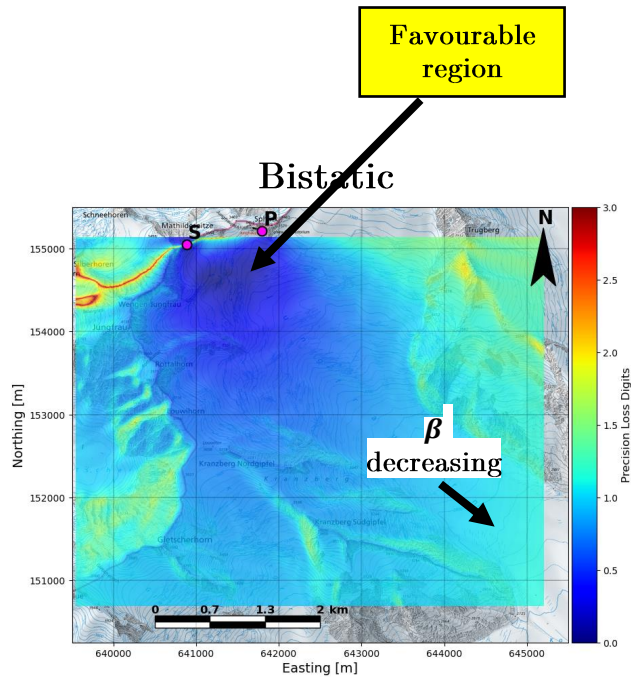


Precision Loss decay in the far range

$$\kappa = \lim_{\delta \rightarrow 0} \sup_{\|\delta x\| \leq \delta} \left( \frac{\|\delta f\|}{\|f\|} / \frac{\|\delta x\|}{\|x\|} \right)$$

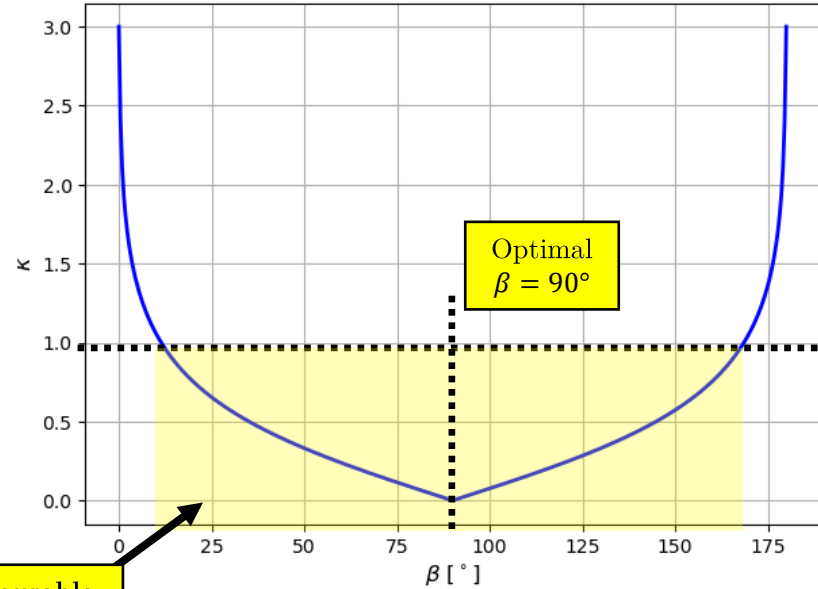
$$C = \log_{10} \kappa(A)$$

# Displacements results: Digit Precision Loss



$$\kappa(A) = \|A\| \cdot \|A^{-1}\|$$

Precision Loss 2D case

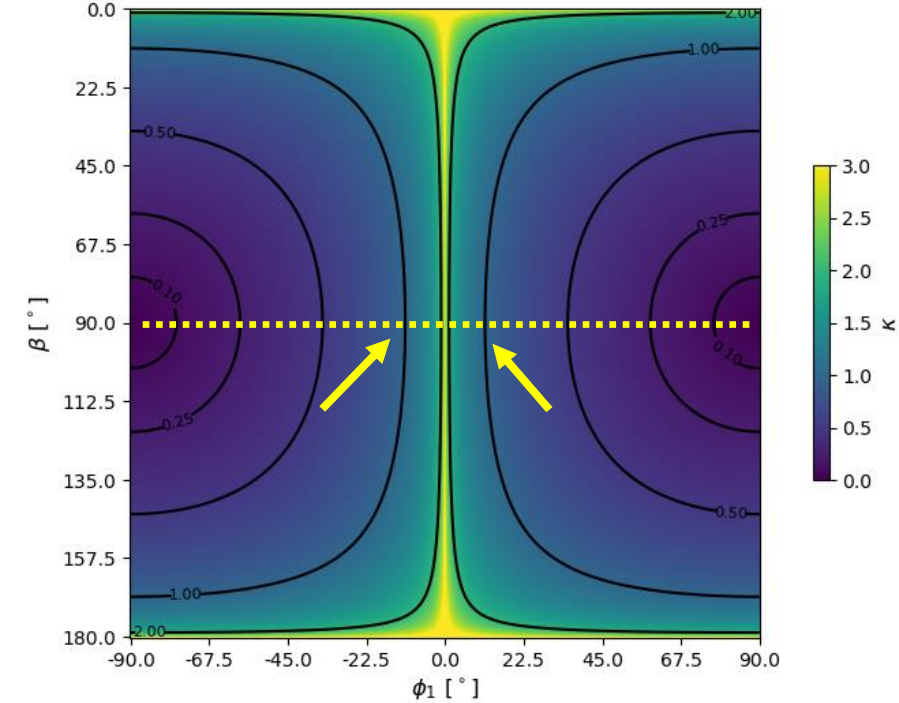


Favourable region

Displacement always contained in LOS plane

Only dependence with bistatic angle  $\beta$ . Limit value at  $\beta \leq 11.31$  deg ( $\kappa \geq 1$ )

Precision Loss 3D case



2D case when  $\varphi_1 = 90^\circ$

Dependence on bistatic angle  $\beta$  and on local elevation angle  $\varphi_1$ . Additional limit on  $\kappa$  when  $\varphi_1 \rightarrow 90$  deg



# Conclusions

## Jungfraujoch Experiment Outcome

### Monostatic-only Approach:

Displacement direction uncoupled from radar observations. Low SNR/geometry only affects magnitude

- Uncertainty low due to high SNR across the scene
- Low precision loss except challenging topography areas (LOS perpendicular to gradient)

### Monostatic-bistatic Approach:

Displacement direction and magnitude presents robustness in the near range, both vulnerable to low SNR

- Uncertainty high due to low SNR (decay proportional to range)
- Precision loss map (specific configuration used) far range decay ( $\beta \rightarrow 0^\circ$ ) with favorable geometry in near range ( $\beta \approx 90^\circ$ )

# References

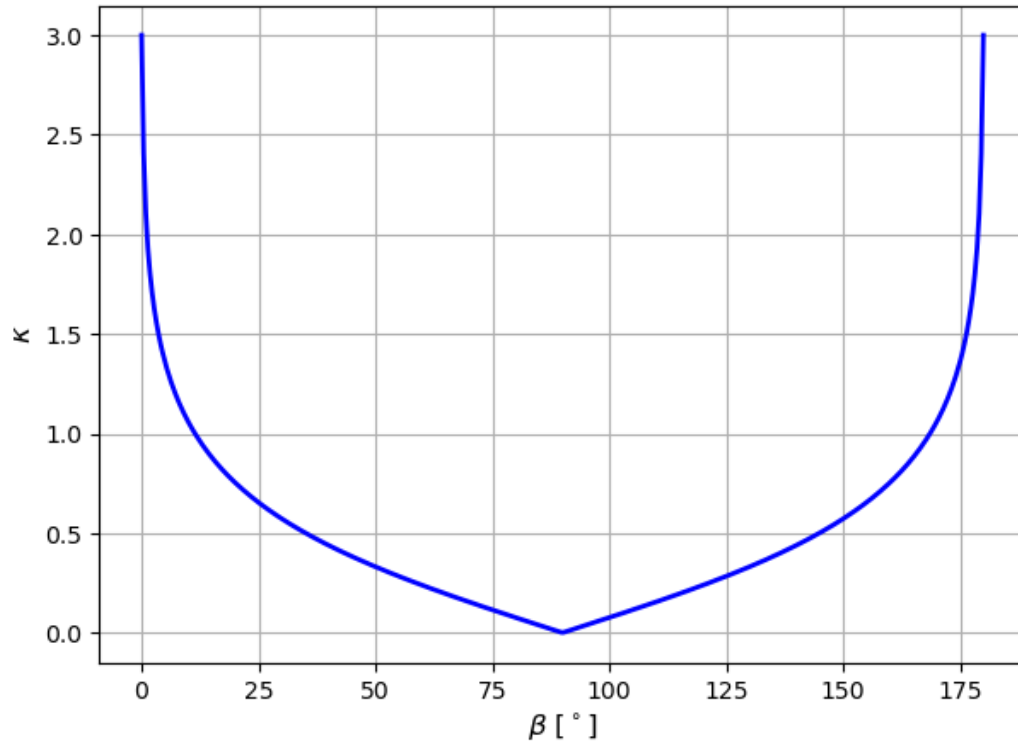
Mas Sanz, E., Stefko, M., Hajnsek, I., (under review) “DEM-assisted 3D reconstruction of Aletsch glacier displacements using monostatic and bistatic differential interferometry”.  
*Manuscript submitted for publication.*



# Extra Slides

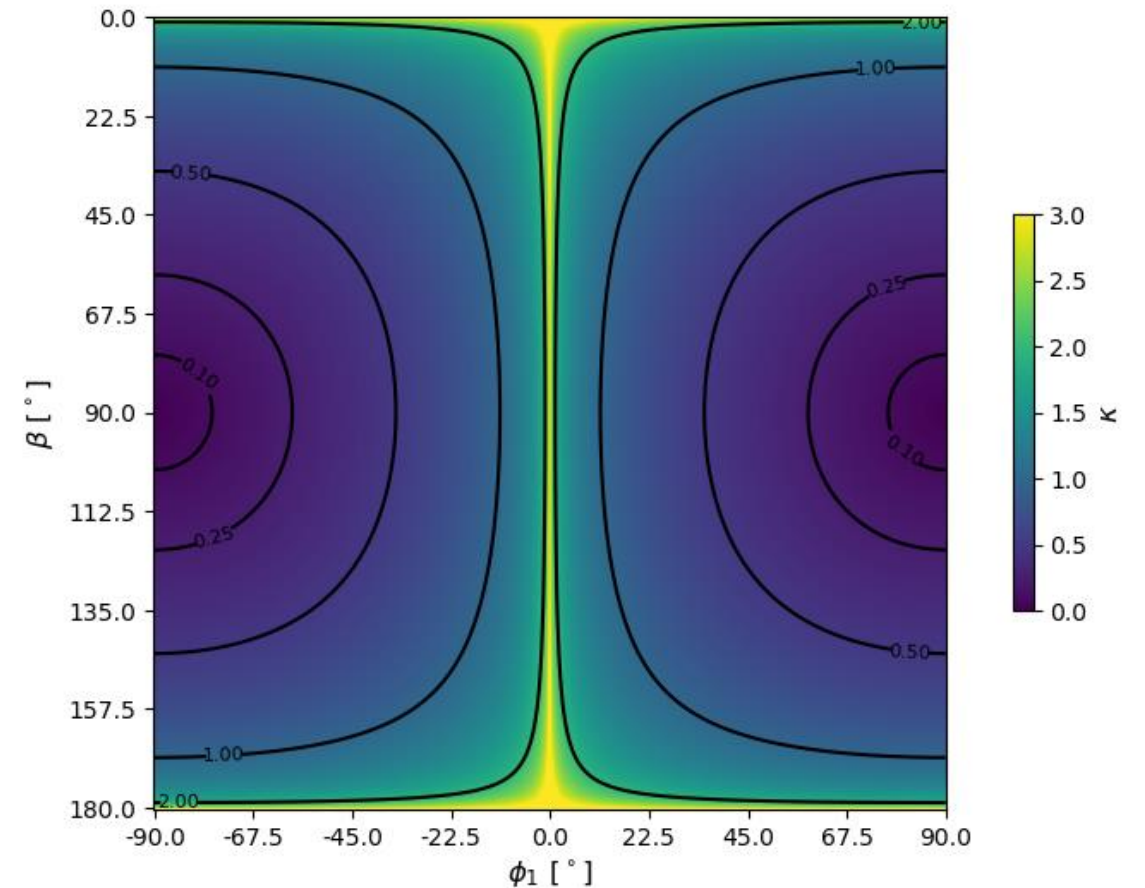
# Displacements results: Digit Precision Loss

Precision Loss 2D case



Only dependence with bistatic angle  $\beta$ . Limit value at  $\beta \leq 11.31$  deg ( $\kappa \leq 1$ )

Precision Loss 3D case



Dependence on bistatic angle  $\beta$  and on local elevation angle  $\phi_1$ . Additional limit on  $\kappa$  when  $\phi_1 \rightarrow 90$  deg



# Displacement Calculation: Monostatic Case

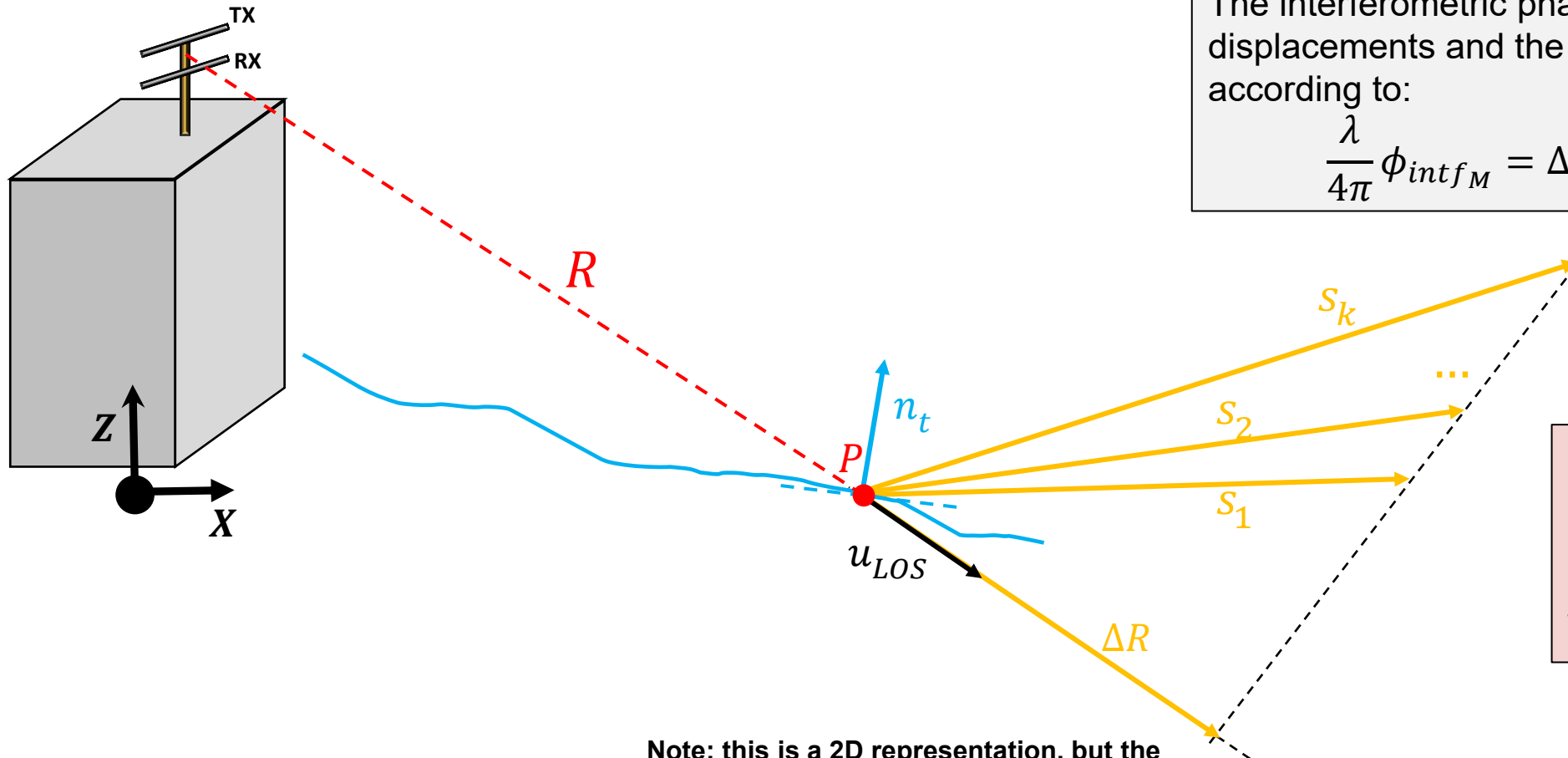
$$R_{TX} = R_{RX}$$

The TX leg is the same as the RX leg!

$$\text{Interferogram [mono]: } \phi_{intf_M} = \frac{4\pi}{\lambda} \Delta R$$

The interferometric phase relates to the LOS displacements and the **true displacements** according to:

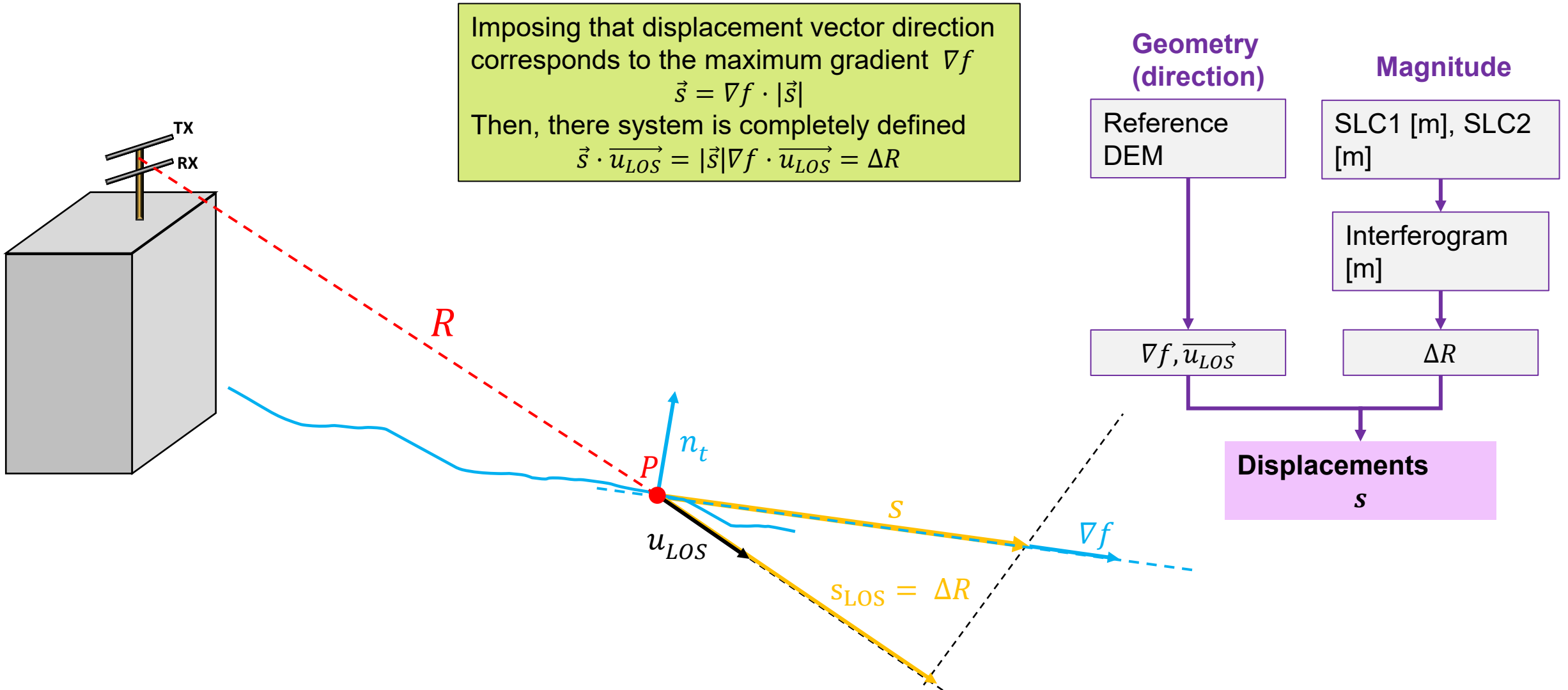
$$\frac{\lambda}{4\pi} \phi_{intf_M} = \Delta R = \vec{s}_k \cdot \vec{u}_{LOS}$$



Which is the value of the true displacements  $\vec{s}$ ?  
There are infinite solutions that satisfy:  
 $\vec{s}_k \cdot \vec{u}_{LOS} = \Delta R$

Note: this is a 2D representation, but the problem is 3D. In a general case  $u_{LOS}$ ,  $n_t$ ,  $s$  are not contained in the same plane

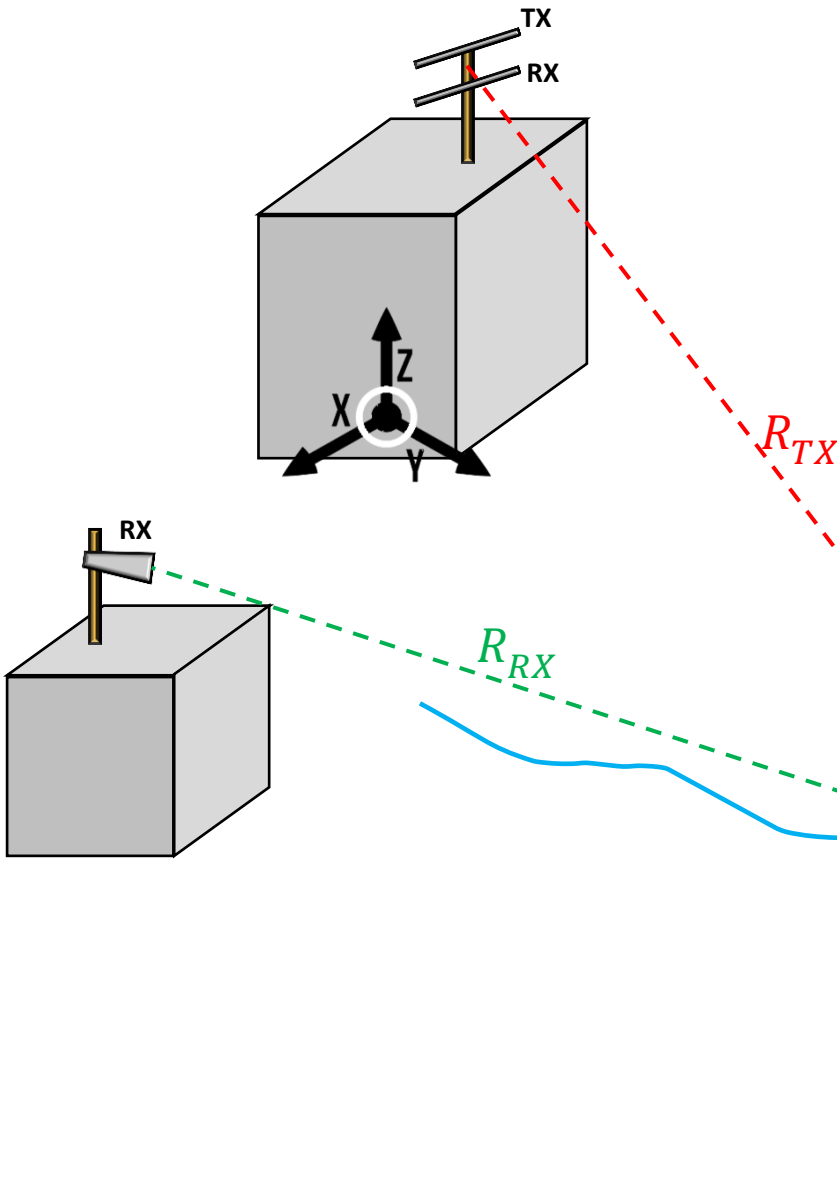
# Displacement Calculation: Monostatic Case





# Displacement Calculation: Bistatic Case

The TX leg and RX leg have different look vectors!



$$\text{Interferogram [b]: } \phi_{intf_B} = \frac{2\pi}{\lambda} (\Delta R_{TX} + \Delta R_{RX})$$

Where  $\frac{2\pi}{\lambda} \Delta R_{TX}$  is equivalent to half of the monostatic interferometric phase

$$\frac{2\pi}{\lambda} \Delta R_{TX} = \frac{\phi_{interf_M}}{2}$$

And the LOS displacements seen from the RX leg are:

$$\phi_{intf_{RX\ leg}} = \phi_{intf_B} - \frac{\phi_{interf_M}}{2}$$

The interferometric phases relate to the LOS displacements and the **true displacements** according to:

$$\frac{\lambda}{4\pi} \phi_{intf_M} = \Delta R_{TX} = \vec{s}_k \cdot \vec{u}_{LOS_M}$$

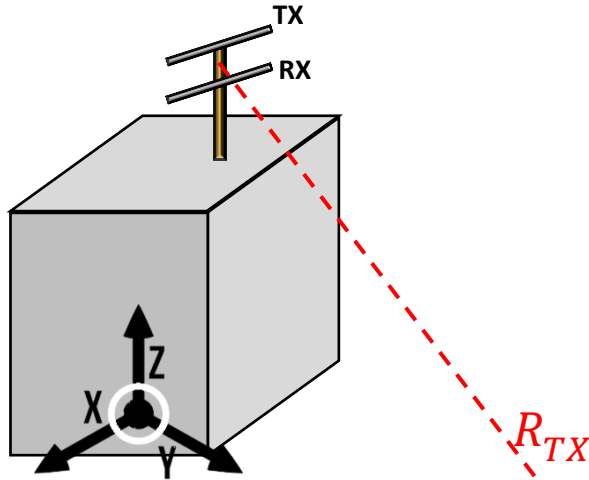
$$\frac{\lambda}{4\pi} \phi_{intf_{RX\ leg}} = \Delta R_{RX} = \vec{s}_k \cdot \vec{u}_{LOS_B}$$

Which is the value of the true displacements  $\vec{s}$  ?

There are infinite solutions that satisfy:  
 $\Delta R_{TX} = \vec{s}_k \cdot \vec{u}_{LOS_M}$  and  $\Delta R_{RX} = \vec{s}_k \cdot \vec{u}_{LOS_B}$

?

# Displacement Calculation: Bistatic Case



Imposing that displacement vector is contained in the plane tangential to the surface. In other words, that the dot-product between the normal of the surface and the true displacements is 0:

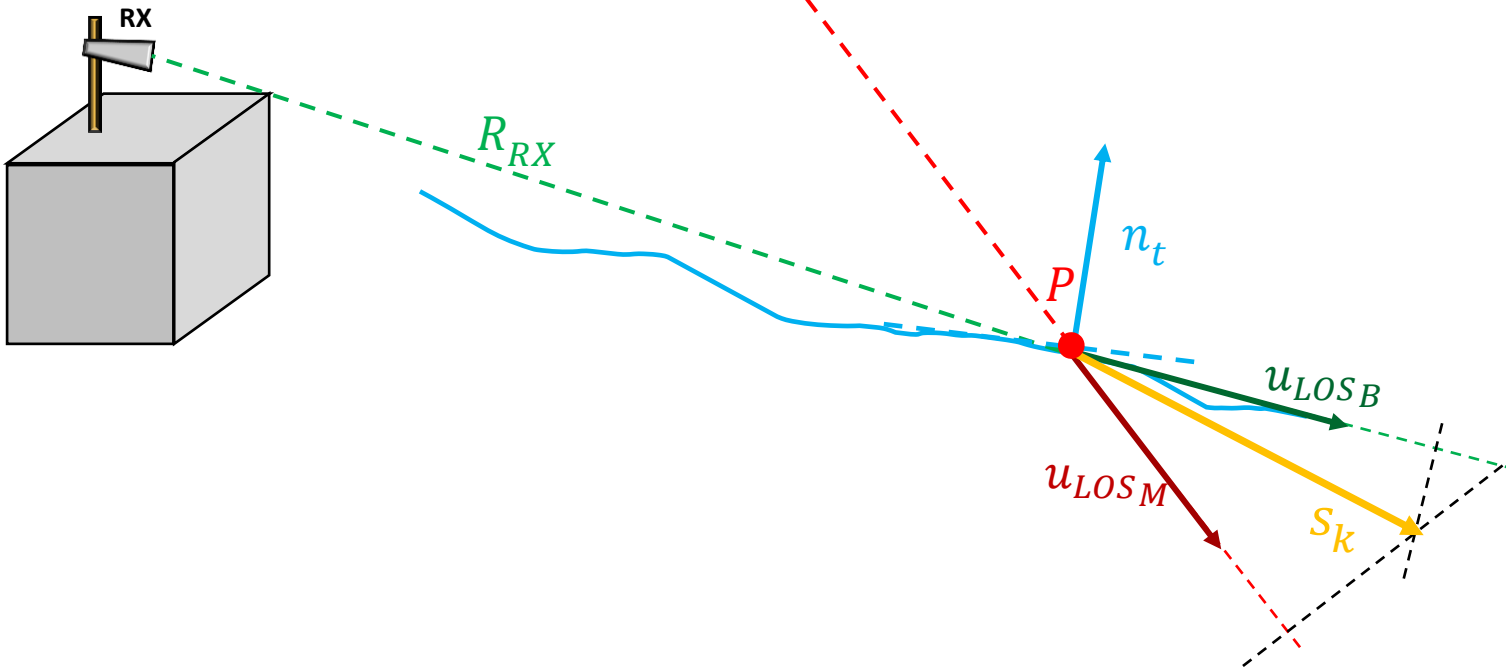
$$\vec{s} \cdot \vec{n}_t = 0$$

Together with equations:

$$\Delta R_{TX} = \vec{s} \cdot \vec{u}_{LOS_M}$$

$$\Delta R_{RX} = \vec{s} \cdot \vec{u}_{LOS_B}$$

Forms an independent system with one solution



**Geometry  
(direction restriction)**

Reference DEM

$\vec{n}_t, \vec{u}_{LOS_M}, \vec{u}_{LOS_B}$

**Magnitude & Direction**

SLC1 [m]  
SLC2 [m]

SLC1 [b]  
SLC2 [b]

Interferogram [m]

Interferogram [b]

$\Delta R_{TX}$

Interferogram  
[RX leg]

$\Delta R_{RX}$

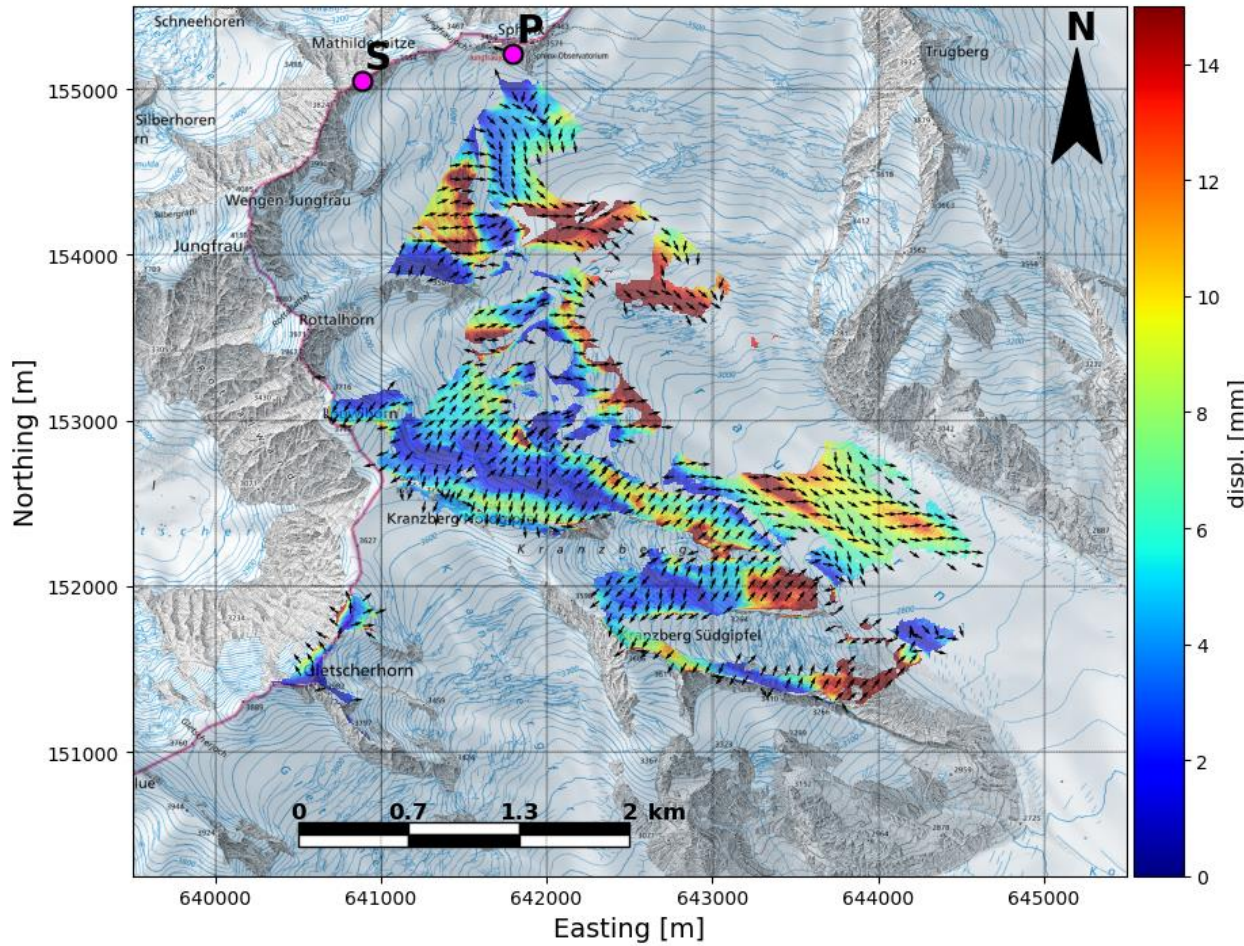
Displacements

$\vec{s}$

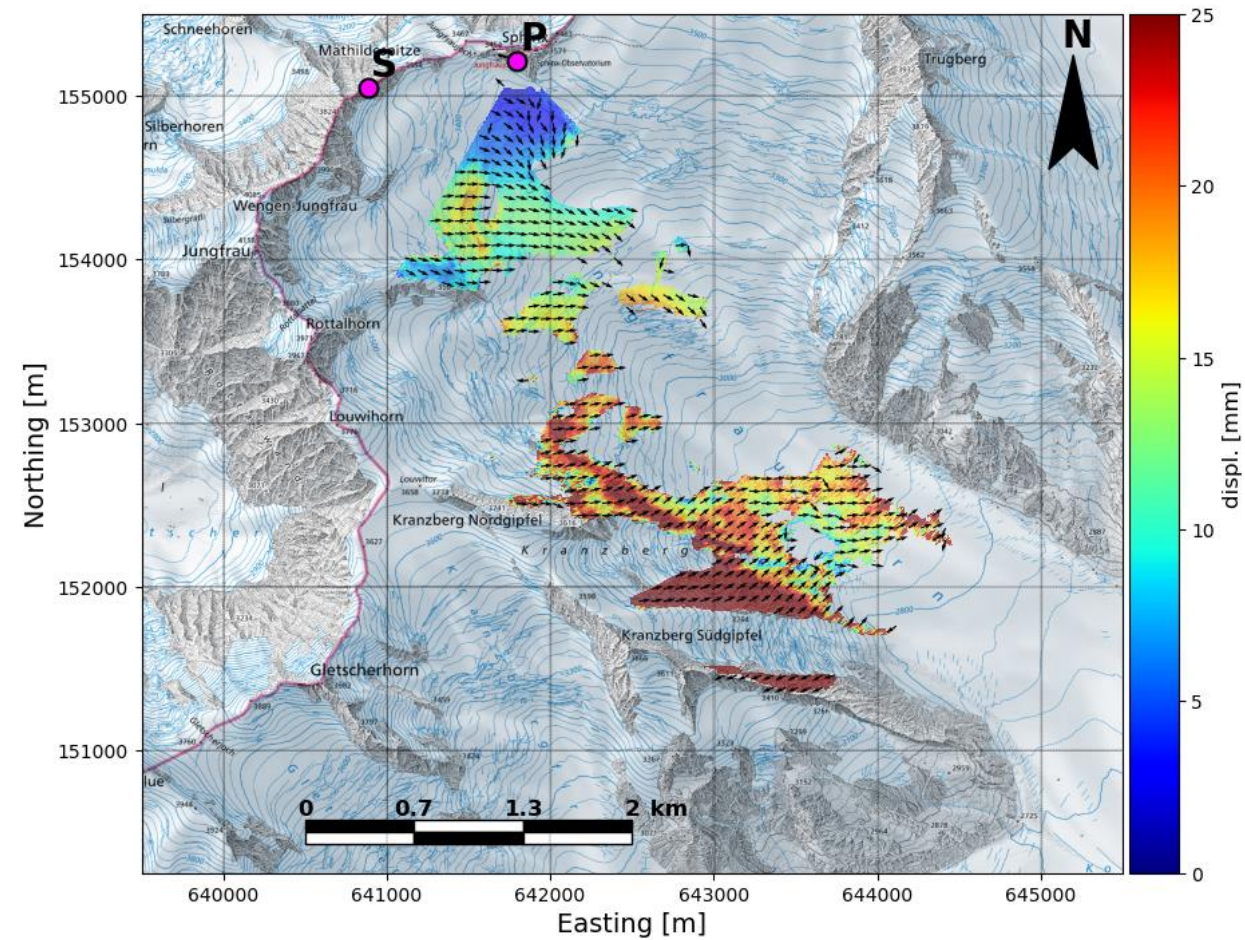


# Displacements results: Displacement Vector Field

Monostatic



Bistatic





# Displacements results: Displacement Vector Field

**Monostatic**

**Bistatic**

ROI 1 (m) // (b)

Mean displ. [mm]: 3.83 // 4.42

Std.dev [mm]: 1.17 // 0.85

Mean  $\phi$ [deg]: -6.42 // -6.15

Std dev.  $\phi$ [deg]: 1.68 // 1.58

Mean  $\theta$ [deg]: 139.11 // 132.21

Std dev.  $\theta$ [deg]: 17.10 // 17.25

ROI 2 (m) // (b)

Mean displ. [mm]: 2.63 // 19.23

Std.dev [mm]: 1.17 // 0.85

Mean  $\phi$ [deg]: -17.21 // -42.59

Std dev.  $\phi$ [deg]: 45.04 // 7.82

Mean  $\theta$ [deg]: -11.51 // 72.76

Std dev.  $\theta$ [deg]: 86.03 // 18.13

ROI 3 (m) // (b)

Mean displ. [mm]: 8.54 // 17.82

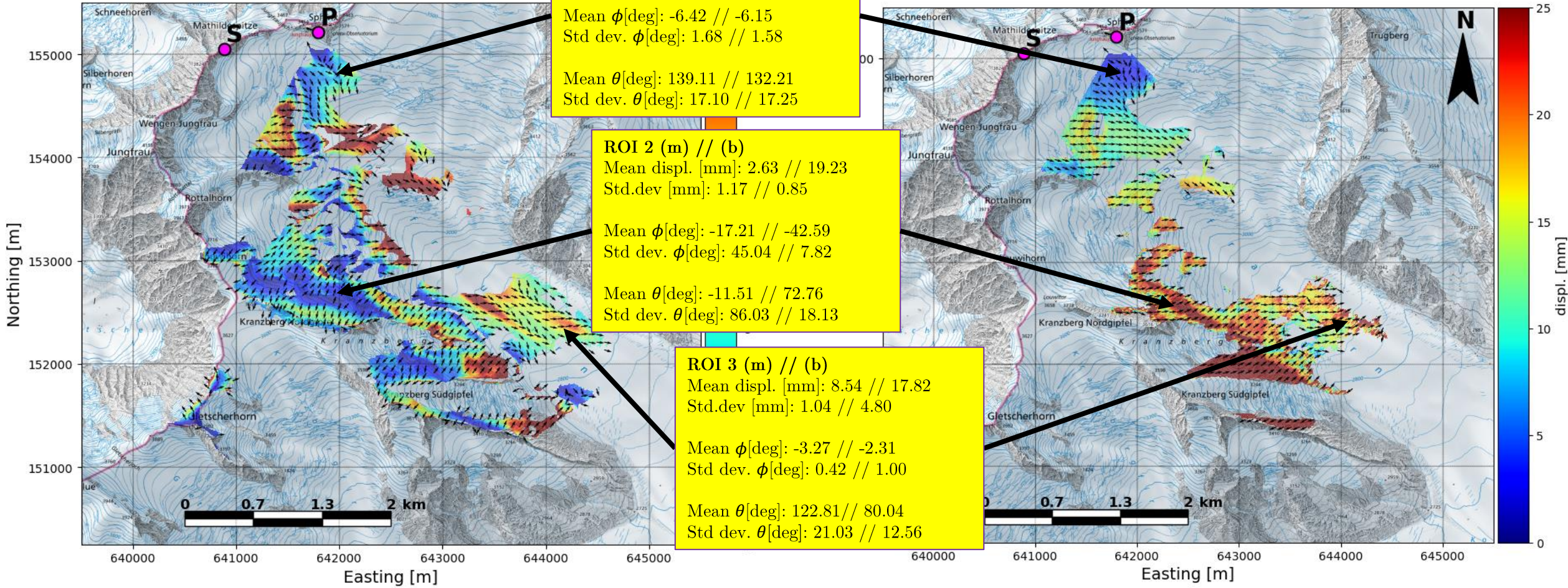
Std.dev [mm]: 1.04 // 4.80

Mean  $\phi$ [deg]: -3.27 // -2.31

Std dev.  $\phi$ [deg]: 0.42 // 1.00

Mean  $\theta$ [deg]: 122.81 // 80.04

Std dev.  $\theta$ [deg]: 21.03 // 12.56





# Processing Pipeline

