



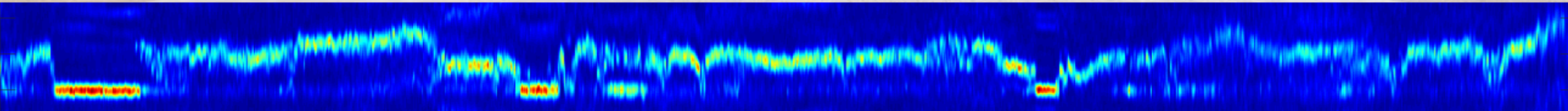
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Multistatic Radar Workshop 2025

Interferometric modelling for FDM MIMO SAR

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Multiple Input Multiple Output (MIMO) technology is a most attractive solution for distributed spaceborne SAR

In simple terms:

- All satellites act both as Tx and Rx
- The data acquired by a given Tx/Rx pair can be thought of as acquired by a **virtual** monostatic SAR placed in the middle (*in the limit of small bistatic angles*)

Real formation



4 real elements

Virtual formation



*9 evenly-spaced
virtual elements*

The underlying assumption in MIMO literature is that the echoes associated with different transmissions can be perfectly separated at the Rx.

⇒ Need for orthogonal waveforms

Time Division Multiplexing (TDM): each transmission occurs at a different time

- High peak power required to compensate for the shorter Tx time
- Time-synchronization required to ensure correct reception of all signals

Code Division Multiplexing (CDM): simultaneous transmission of orthogonal sequences

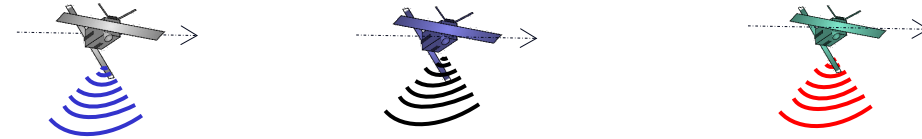
- Orthogonality can only be granted within a window shorter than Tx time

⇒ Ok for short range Radars, not fit for spaceborne SARs

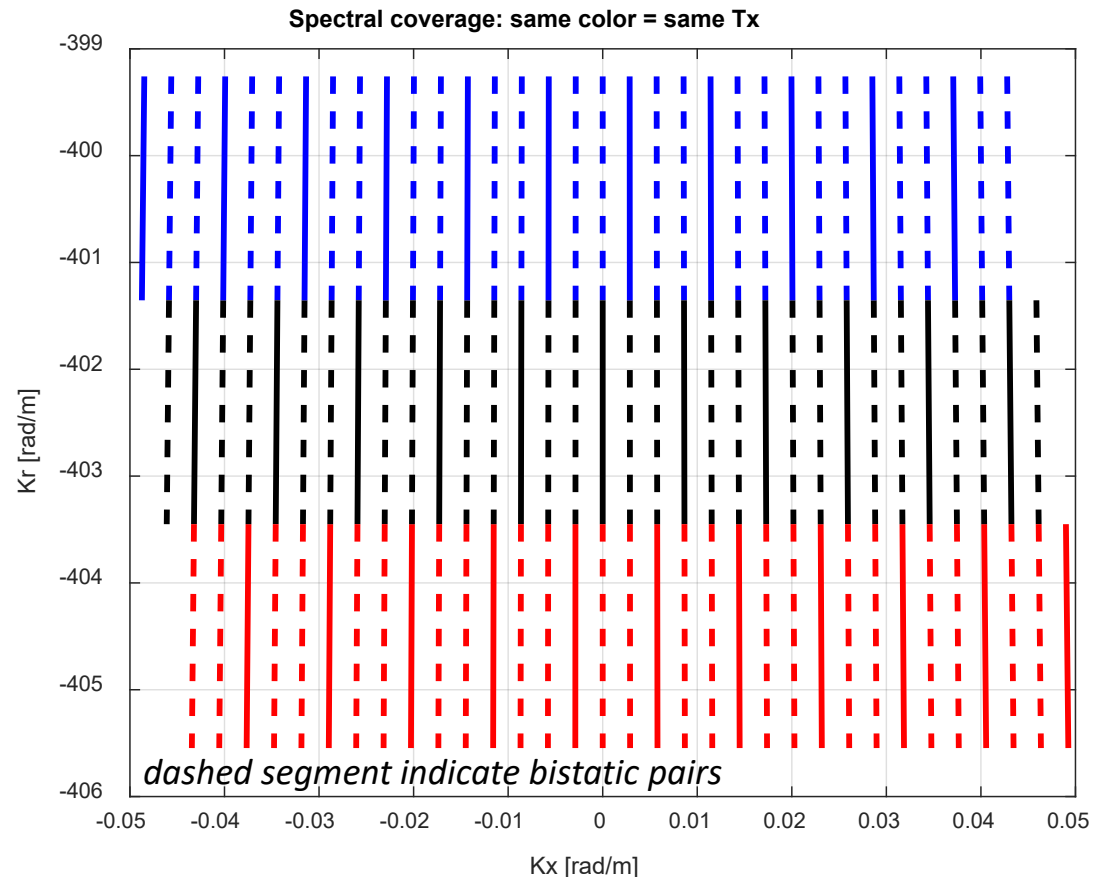
Frequency Division Multiplexing (FDM): simultaneous transmission in different frequency bands

- Nearly perfect orthogonality
- Simple hardware & design
- Versatile, fit for both AT and XT formations

High Resolution Wide Swath by AT formations



Wavenumber coverage



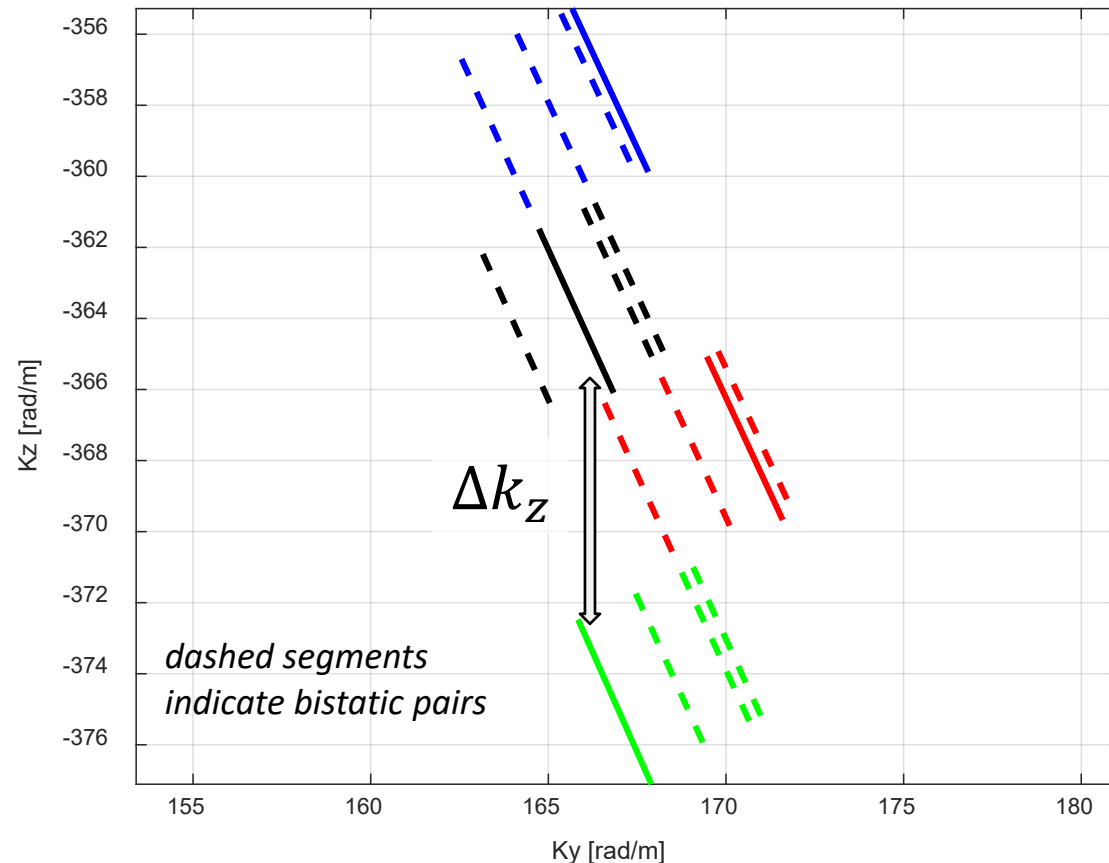
- Tx occur at lower PRF to abate range ambiguities
- Equivalent high PRF ensured by bistatic Rx
- Large bandwidth achieved by multiple Tx in different frequency bands

Multi-baseline InSAR/TomoSAR by XT formations

Wavenumber coverage



Spectral coverage: same color = same Tx

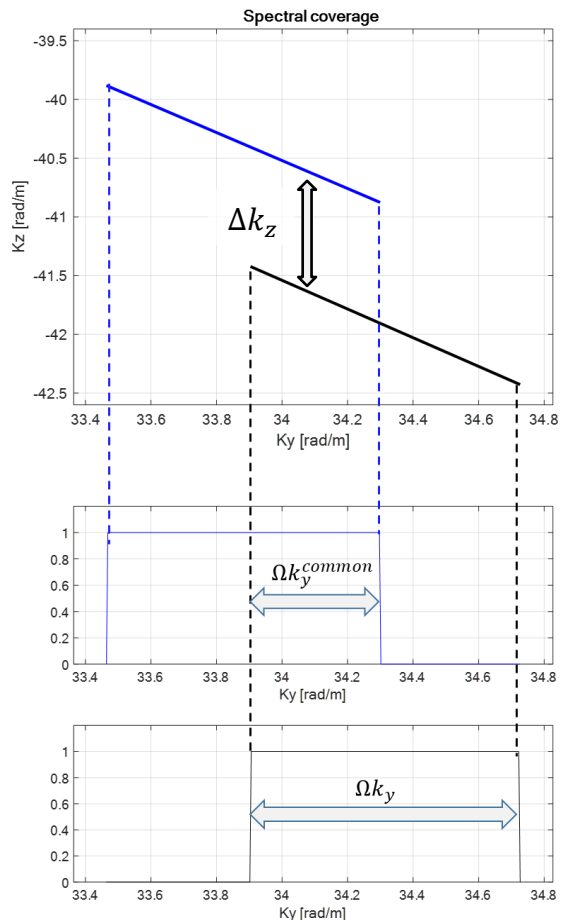
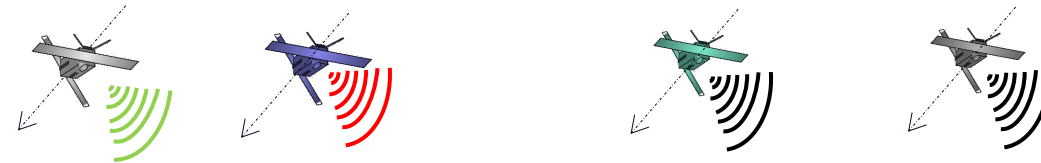


The interferometric vertical wavenumber Δk_z is obtained as the vertical separation between any two segments representing mono- or bistatic acquisitions

- Frequency diversity creates large Δk_z for monostatic pairs
- ⇒ Fine vertical resolution
- Bistatic pairs guarantee low Δk_z
- ⇒ Large height of ambiguity

What about coherence?

Multi-baseline InSAR/TomoSAR by XT formations



Flat terrain decorrelation in any two images is accounted for by taking the projection of individual wavenumber segments onto the k_y axis

$$\gamma = \frac{\Omega k_y^{common}}{\Omega k_y}$$

Multi-baseline InSAR/TomoSAR by XT formations

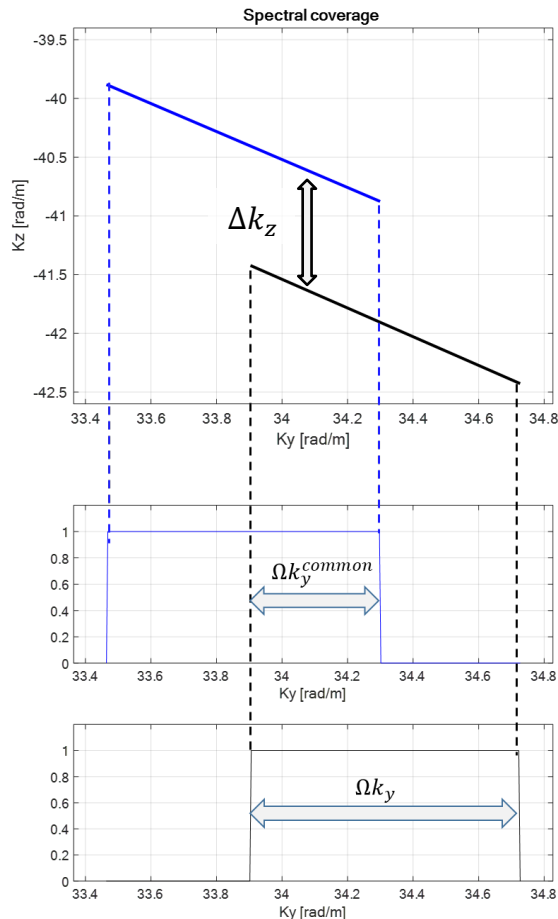


Flat terrain decorrelation in any two images is accounted for by taking the spectral overlap of individual wavenumber segments along the k_y axis

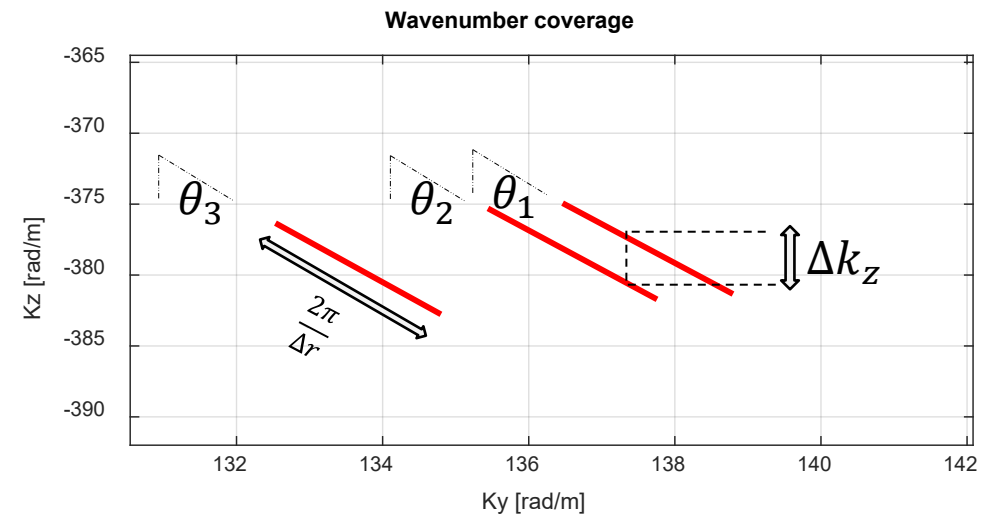
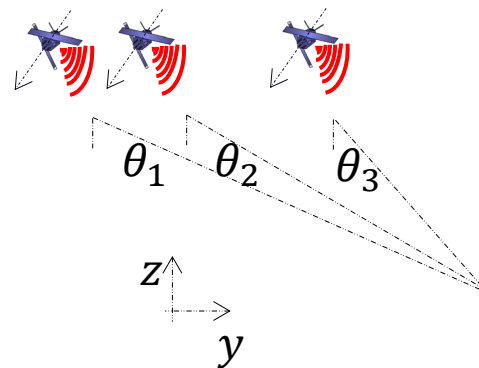
$$\gamma = \frac{\Omega k_y^{common}}{\Omega k_y}$$

In traditional multi-baseline single-platform SARs, spectral overlap and interferometric wavenumbers are inherently linked

⇒ High coherence implies coarse vertical resolution, and viceversa



Single-frequency monostatic SAR



Multi-baseline InSAR/TomoSAR by XT formations

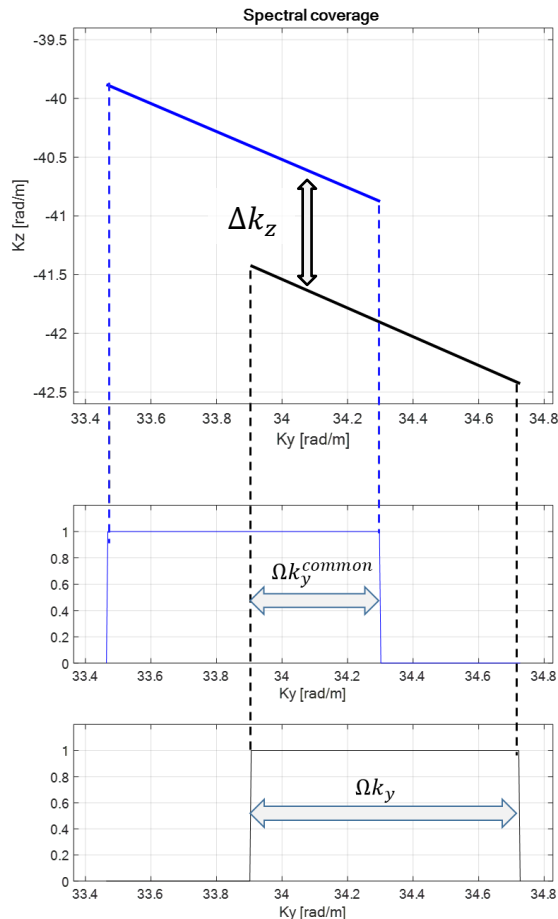


Flat terrain decorrelation in any two images is accounted for by taking the spectral overlap of individual wavenumber segments along the k_y axis

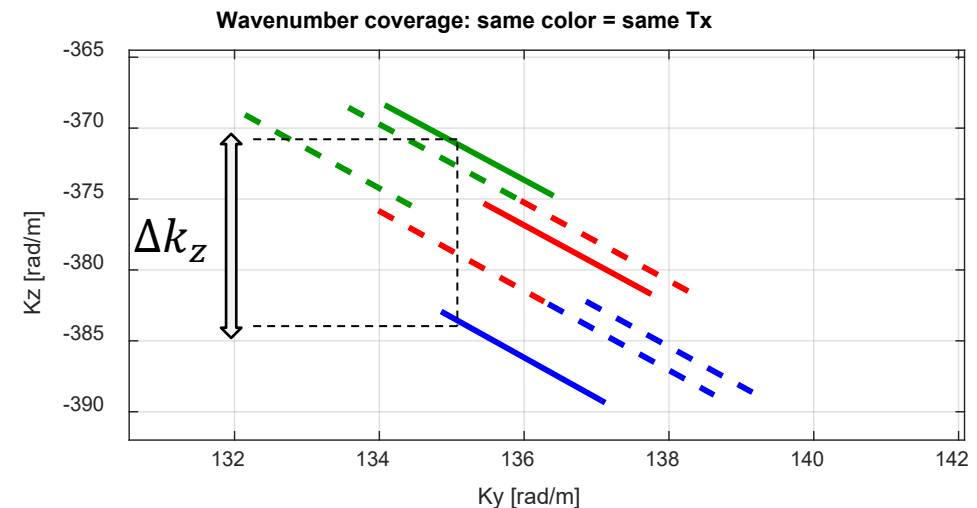
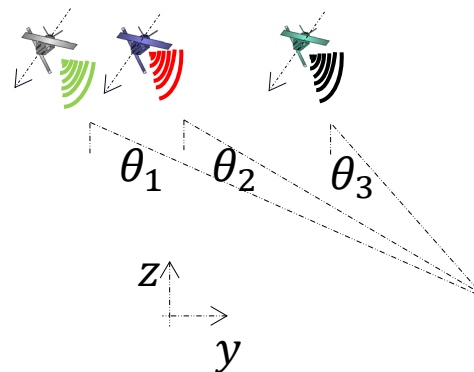
$$\gamma = \frac{\Omega k_y^{common}}{\Omega k_y}$$

Not so for FDM MIMO SAR

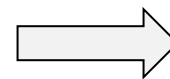
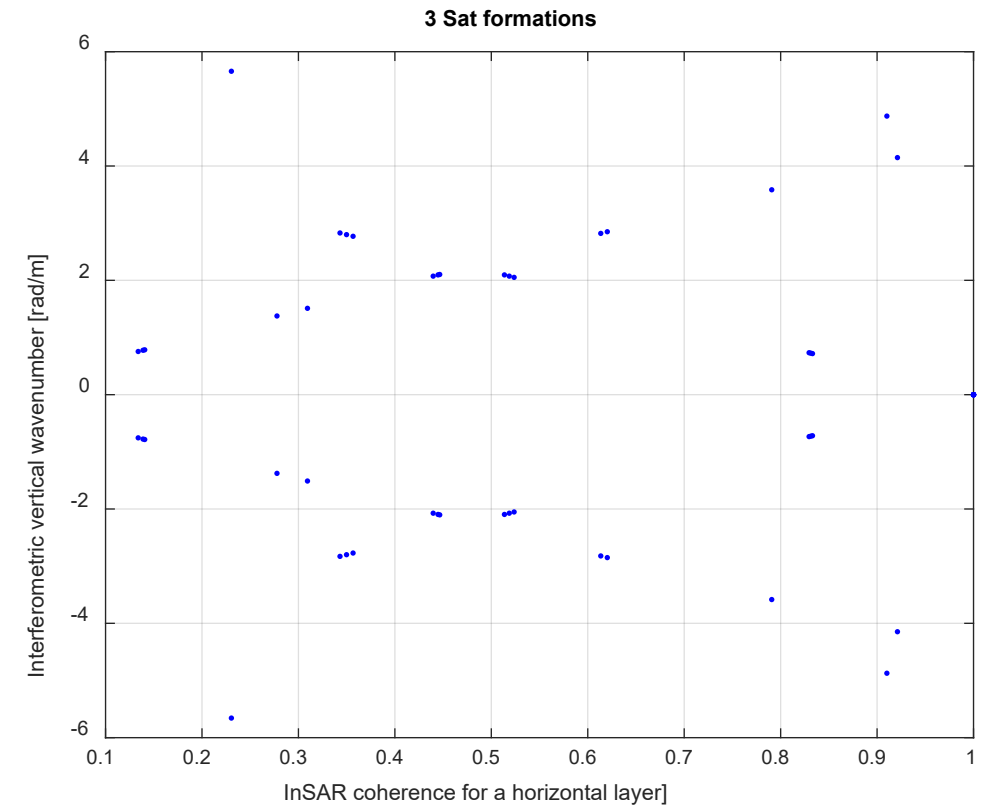
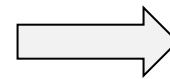
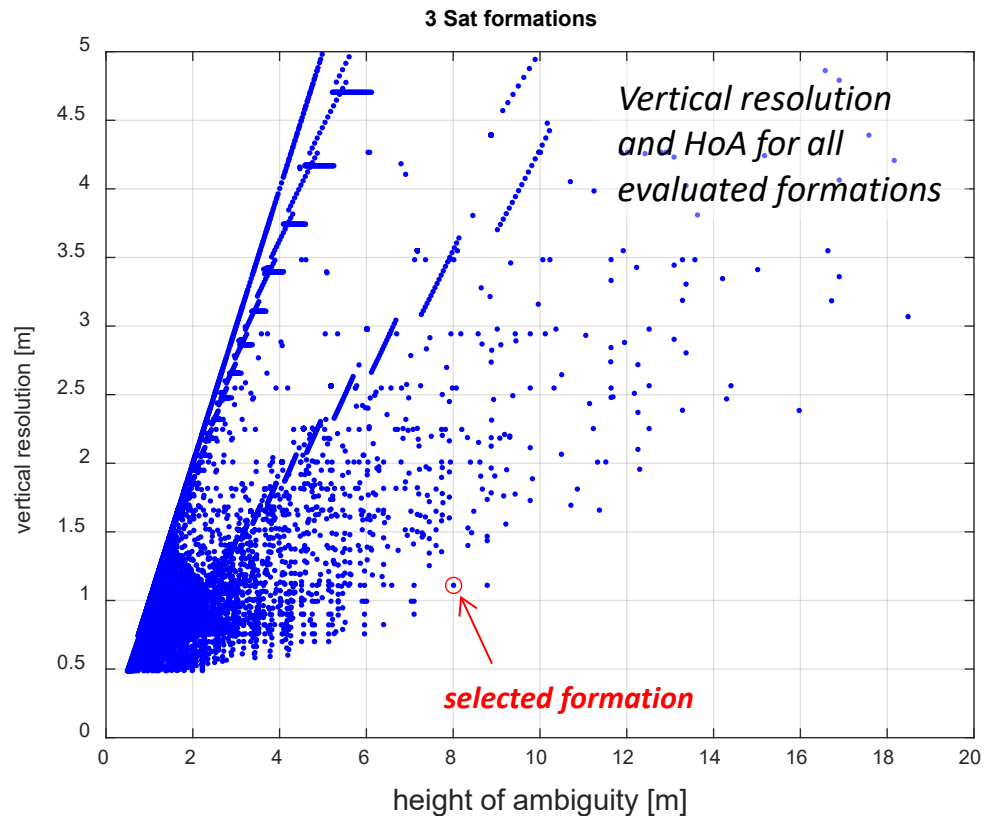
⇒ High coherence at fine vertical resolution is possible



FDM MIMO SAR



Example: design of a three-satellite X-Band FDM MIMO formation for snowpack tomography



N	Optimal deployment (°)	z_{amb} (m)	ρ_z (m)	N_{eff}
3	21.00 20.79 20.71	8.0	1.1	8

- Vertical resolution and HoA equivalent to 8 simultaneous monostatic passes
- Higher coherence for high interferometric wavenumbers

InSAR modelling: covariance matrix for multi-layered media on a sloping terrain

$$\gamma_{nm} = \gamma_{nm}^{layer} \underbrace{\frac{\sum_{k=1}^{K_s} \sigma_k^2 e^{i(\Delta k_z(n) - \Delta k_z(m)) z_k}}{\sum_{k=1}^{K_s} \sigma_k^2}}_{\gamma_{nm}^{vol}}$$

σ_k^2 : intensity of the k -th layer

z_k : height of the k -th layer

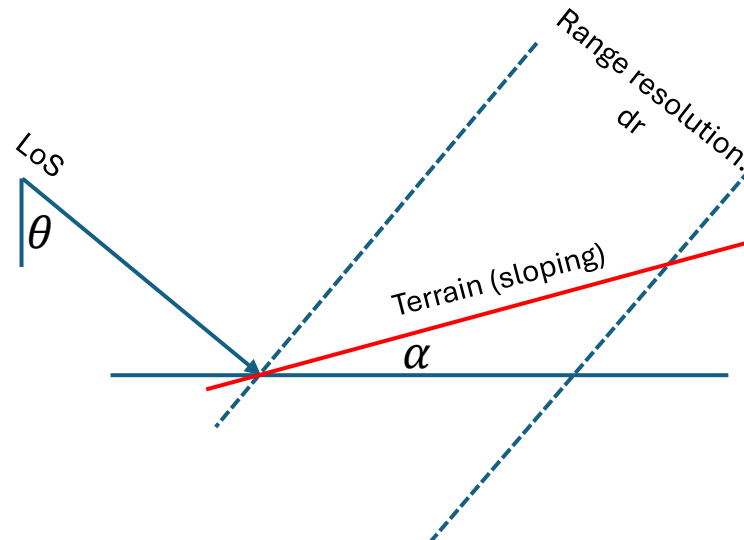
$$\gamma_{nm}^{layer} = 1 - |f_{nm} \Delta y|$$

$$f_y = + \frac{f_{Tx}}{c} (\sin(\theta_{Rx}) + \sin(\theta_{Tx}))$$

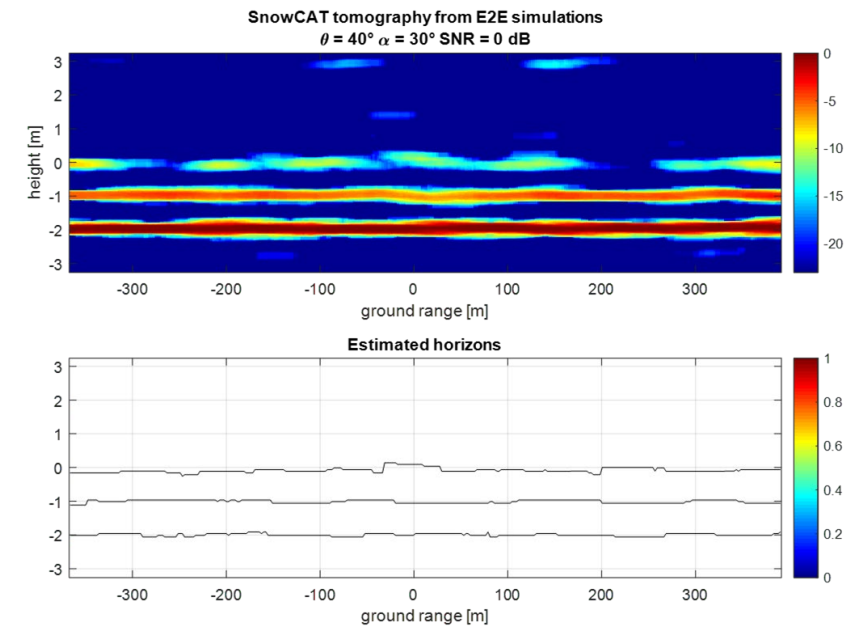
$$f_z = - \frac{f_{Tx}}{c} (\cos(\theta_{Rx}) + \cos(\theta_{Tx}))$$

$$f_{nm} = \frac{f_y(n) - f_y(m) + (f_z(n) - f_z(m)) \tan(\alpha)}{1}$$

$$\Delta y = \frac{c}{2B_{Tx} \sin(\theta) - \cos(\theta) \tan(\alpha)}$$



Inversion tested through E2E simulations



MIMO SAR allows for expanding the data space, enabling different applications, including HRWS, single pass tomography, InSAR, and others...

Frequency Division Multiplexing has some interesting advantages:

- Simple Hardware
- Simple Design
- Nearly perfect orthogonality
- For XT formations, decoupling of terrain decorrelation and interferometric wavenumber opens the way to new possibilities for accurate parameter retrieval